MATHS QUESTION PAPER

Time : 2 Hrs. Max. Marks : 40

Q. 1 (A) Attempt any TWO of the following : [8]

(i) a, b, c are three non-coplanar vectors.

If $\bar{p} = \dfrac{\bar{b} \times \bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]}$, $\bar{q} = \dfrac{\bar{c} \times \bar{a}}{[\bar{a}\ \bar{b}\ \bar{c}]}$, $\bar{r} = \dfrac{\bar{a} \times \bar{b}}{[\bar{a}\ \bar{b}\ \bar{c}]}$

then prove that $(\bar{a} + \bar{b}) \cdot \bar{p} + (\bar{b} + \bar{c}) \cdot \bar{q} + (\bar{c} + \bar{a}) \cdot \bar{r} = 3$. (3)

(ii) Using vector method, prove that the diagonals of a parallelogram bisect each other and conversely. (3)

(iii) If $\bar{a}$ and $\bar{b}$ are non-zero, non-collinear vectors, then prove that any vector $\bar{r}$ coplanar with vectors $\bar{a}$ and $\bar{b}$ can be uniquely expressed as the linear combination $x\bar{a} + y\bar{b}$, for non-zero scalers x and y. (3)

(B) Attempt any ONE of the following :

(i) If θ is the acute angle between the lines $3x^2 + 4xy + by^2 = 0$ and $\tan\theta = \frac{1}{2}$, find b. (2)

(ii) Find the equation of a circle concentric with the circle $x^2 + y^2 + 2x + 2y + 1 = 0$ having radius 3. (2)

Q. 2 (A) (a) Attempt any ONE of the following : [8]

(i) Find the equation of the locus of a point the tangents from which to the parabola $y^2 = 8x$ are such that $\cot\theta_1 + \cot\theta_2 = 3$, where θ_1 and θ_2 are inclinations of tangents. (3)

(iii) Find the equation of locus of the point, the tangents from which to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ are perpendicular to each other. (3)

(b) Attempt any ONE of the following :

(i) Show that the lines $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-1}{-1}$ and $\frac{x-4}{2} = \frac{y-2}{-1} = \frac{z-4}{1}$ intersect each other and find the co-ordinates of their points of intersection. (3)

(ii) If the perpendiculars drawn from (2, 4, 5) on yz and zx planes meet them in L and M respectively, find the equation of the plane OLM where O is the origin. (3)

(B) Attempt any ONE of the following :

(i) If $\bar{a}, \bar{b}, \bar{c}$ are position vectors of points A, B and C respectively such that $3\bar{c} + \bar{a} = 4\bar{b}$, find the ratio in which C divides Seg. AB. (2)

(ii) If the position vectors of two vertices of a triangle are $5\bar{i} + 4\bar{j} + 2\bar{k}$ and $4\bar{i} + 3\bar{j} + 3\bar{k}$ and centroide of the triangle is at origin, find the third vertex of the triangle. (2)

Q. 3 (A) (a) Attempt any ONE of the following : [8]

(i) Find the joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $2x^2 + 7xy + 3y^2 = 0$. (3)

(ii) Find the equation of the circle touching both the axes and passing through the point (–9, 8). (3)

(b) Attempt any ONE of the following :

(i) If $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$. find $P(A \cup B)$,

when (a) A and B are mutually exclusive events.

(b) A and B are independent events. (3)

(ii) Two cards are drawn from a pack of 52 cards. If X = number of red cards drawn, find the probability density function of X. (3)

(B) Attempt any ONE of the following:

(i) If the line $4x - 3y + k = 0$ touches the ellipse $5x^2 + 9y^2 = 45$, then find the value of 'k'. (2)

(ii) For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ find the eccentricity and length of latus rectum. (2)

Q. 4 (A) Attempt any TWO of the following : **[8]**

(i) Write the converse, inverse and contrapositive of the statement "If a function is differentiable then it is continuous." **(3)**

(ii) If the statements p and q are true, r and s are false, determine the truth values of the following statement patterns : **(3)**

(a) $p \vee (q \wedge r)$ (b) $(p \wedge \sim r) \wedge (\sim q \vee s)$ (c) $(p \rightarrow q) \vee (r \leftrightarrow s)$

(iii) Using the truth table, show that **(3)**

$\sim p \ (\leftrightarrow q) \leftrightarrow [p \wedge \sim q)] \vee [q \wedge (\sim p)]$ is a tautology.

(B) Attempt any ONE of the following :

(i) A company manufactures two types of show pieces A and B made up of plywood. Show piece of type A requires 5 minutes for cutting and 10 minutes for assembling show piece of type B needs 8 minutes for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The profit is Rs. 5 for each piece of type A and Rs. 6 for each piece of type B. Formulate this problem as a L.P.P. to maximize the profit. **(2)**

(ii) Solve the following L.P.P. graphically

Minimize $Z = 6x + 7y$

Subject to $x + 3y \geq 3$

$2x + y \geq 2$

$x \geq 0, y \geq 0$ **(2)**

Q. 5 (A) (a) Attempt any ONE of the following : **[8]**

(i) Find the inverse of the matrix $A = \begin{bmatrix} 6 & 2 & 2 \\ -3 & 7 & 1 \\ 3 & 5 & -1 \end{bmatrix}$ by using the method of adjoint. **(3)**

(ii) Solve the following equations using reduction method

$x + y + z = 3, 3x - 2y + 3z = 4, 5x + 5y + z = 11.$ **(3)**

(b) Attempt any ONE of the following :

(i) If θ is the measure of the acute angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$, then show that

$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$ **(3)**

(ii) Derive the equation of the director circle of the circle $x^2 + y^2 = a^2$. **(3)**

(B) Attempt any ONE of the following :

(i) Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to the line $3y = x + 4$. **(2)**

(ii) If P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and S and S' are its foci, prove that $|SP - S'P| = 2a$. **(2)**