MATHS QUESTION PAPER

Time : 2 Hrs. **Max. Marks : 40**

Q. 1 (A) Attempt any Two of the following : [8]

(i) Evaluate : $\lim_{x \to 0} \frac{(3^x - 1)^2}{\tan x \cdot \log (1 + x)}$ (3)

(ii) Evaluate, $\lim_{x \to \infty} \sqrt{x}\ (\sqrt{x+5} - \sqrt{x})$ (3)

(iii) Discuss the continuity of the function f (x) on its domain, where,

$$f(x) = x^2 + 4, \text{ for } 0 \le x \le 2$$
$$= 3x + 2, \text{ for } 2 < x < 4$$
$$= x^2 + 1, \text{ for } 4 \le x \le 6$$ (3)

(B) Attempt any One of the following :

(i) Form the differential equation by eliminating the arbitrary constant 'a' from the relation $(x - a)^2 + y^2 = 1$. (2)

(ii) Solve the differential equation $\cos^2 y\, dx - \text{cosec}\, x\, dy = 0$ (2)

Q. 2 (A) (a) Attempt any One of the following :

(i) Show that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$ (3)

(ii) If $f(x) = 2^x$, show that f (x), $\Delta f(x)$, $\Delta^2 f(x)$, ... $\Delta^n f(x)$ are in geometric progression. (3)

(b) Attempt any One of the following :

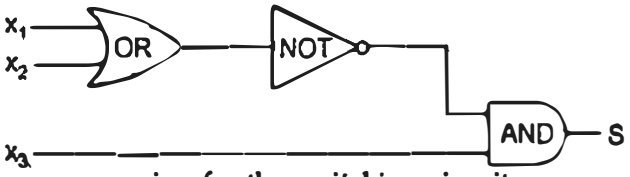
(i) Find the particular solution of the differential equation :

$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$, when $x = \frac{\pi}{4}$, $y = 0$. (3)

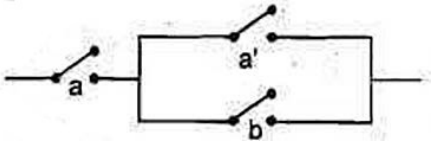
(ii) Solve the differential equation $(x + y)\frac{dy}{dx} = y$. (3)

(B) Attempt any One of the following :

(i) If the inputs are $x_1 = 1$, $x_2 = 0$ and $x_3 = 1$, find output 'S' from the following circuit. (2)



(ii) Find the Boolean expression for the switching circuit



Also draw an equivalent circuit. (2)

Q. 3 (A) Attempt any Two of the following : [8]

(i) If $y = (\sin^{-1} x)^2$, show that

$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ (3)

(ii) Find $\frac{dy}{dx}$, if $y = (\tan x)^x + (4)^{\sin x}$ (3)

(iii) Find the approximate value of $f(x) = 2x^3 + 7x^2 - 2x + 3$, when $x = 2.002$. (3)

(B) Attempt any One of the following :

(i) Write the duals of the following statements

(a) $(x' + 0) \cdot (x \cdot 1) = 0$

(b) $(x \cdot y') + y = x + y$ (2)

(ii) If 'B' is a Boolean algebra, for all $x, y \in B$ prove that

(a) $x + (x' \cdot y) = x + y$

(b) $x \cdot (x' + y) = x \cdot y$ (2)

Q. 4 (A) (a) Attempt any One of the following :

(i) If $y = f(x)$ is a differentiable function of x such that the inverse function $x = f^{-1}(y)$ is define then prove that

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}, \text{ where } \frac{dy}{dx} \neq 0 \qquad (3)$$

(ii) If x and y are differentiable functions of t so that y is a function of x, then prove that

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}, \text{ where } \frac{dx}{dt} \neq 0 \qquad (3)$$

(b) Attempt any One of the following :

(i) If u and v are functions of x, then prove that

$$\int u \cdot v \, dx = u \int v \, dx - \int \left[\int v \, dx \right] \frac{du}{dx} \, dx. \qquad (3)$$

(ii) Prove that

$$\int_0^{2a} f(x) \, dx = \int_0^{a} f(x) \, dx + \int_0^{a} f(2a - x) \, dx \qquad (3)$$

(B) Attempt any One of the following :

(i) Evaluate : $\int \frac{dx}{\sqrt{x^2 + 6x + 5}}$ (2)

(ii) Evaluate : $\int \frac{(\sin^{-1} x)^3}{\sqrt{1 - x^2}} dx$ (2)

Q. 5 (A) (a) Attempt any One of the following : [8]

(i) Evaluate : $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + a)}}$ (3)

(ii) Evaluate : $\int \frac{\log x \, dx}{(1 + \log x)^2}$ (3)

(b) Attempt any One of the following :

(i) Evaluate $\int_0^{\pi/4} \frac{\sec^2 x \, dx}{(1 + \tan x)(2 + \tan x)}$ (3)

(ii) Show that :

$$\int_1^3 \frac{\sqrt[3]{x + 5}}{\sqrt[3]{x + 5} + \sqrt[3]{9 - x}} \, dx = 1 \qquad (3)$$

(B) Attempt any One of the following :

(i) If $y = \sin(x + y)$, find $\frac{dy}{dx}$. (2)

(ii) If $y = \cos^{-1}(4x^3 - 3x)$, find $\frac{dy}{dx}$. (2)