MATHS QUESTION PAPER

Time : 2 Hrs. **Max. Marks : 40**

Q. 1 (A) Attempt any Two of the following :

(i) Discuss the continuity of the following function at $x = 1$. **(3)**

$$f(x) = \begin{cases} \dfrac{1}{1-x} - \dfrac{3}{1-x^3} + \dfrac{7}{4}, & \text{when } x < 1 \\ \dfrac{3}{4}, & \text{when } x = 1 \\ \dfrac{\log x}{x-1} - \dfrac{1}{4}, & \text{when } x > 1 \end{cases}$$

(ii) Evaluate : $\lim\limits_{x \to 0} \dfrac{(4^x - 1)(1 - \cos 4x)}{3 \sin x - \sin 3x}$ **(3)**

(iii) If $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$, when $x \neq \dfrac{\pi}{3}$ **(3)**

$= \dfrac{4}{3}$, when $x = \dfrac{\pi}{3}$

discuss the continuity of the function at $x = \dfrac{\pi}{3}$.

(B) Attempt any One of the following :

(i) If $f'(x) = 4x^3 - 3x^2 + 2x + k$, find $f(x)$ given that $f(0) = 1$ and $f(1) = 4$. **(2)**

(ii) Evaluate : $\displaystyle\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x}\, dx$ **(2)**

Q. 2 (A) Attempt any Two of the following :

(i) Find the derivative of $x \sin x$ with reference to x by first principle. **(3)**

(ii) If $(x^2 + y)^{17} = x^8 y^{13}$, prove that $\dfrac{dy}{dx} = \dfrac{2y}{x}$. **(3)**

(iii) A man of 2 metres height walks at a uniform speed of 6 km/hr away from a lamp post of 6 metres high. Find the rate at which the length of his shadow increases. **(3)**

(B) Attempt any One of the following :

(i) Form the differential equation by eliminating the arbitrary constants from the equation $y = a \cos(\log x) + b \sin(\log x)$. **(2)**

(ii) Verify that $y = ae^{-bx}$ is a solution of $\dfrac{d^2y}{dx^2} = \dfrac{1}{y}\left(\dfrac{dy}{dx}\right)^2$ **(2)**

Q. 3 (A) (a) Attempt any One of the following :

(i) Evaluate : $\displaystyle\int \frac{\log x}{(1 + \log x)^2}\, dx$ **(3)**

(ii) Evaluate : $\displaystyle\int \frac{dx}{\cos x(2 + \sin x)}$ **(3)**

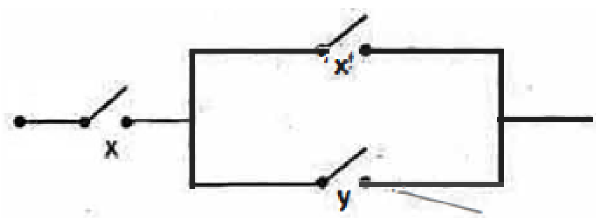
(b) Attempt any One of the following :

(i) Evaluate : $\displaystyle\int_0^3 x^2 (3 - x)^{1/2}\, dx$ **(3)**

(ii) Find the volume of a cone of height 'h' and base radius 'r'. **(3)**

(B) Attempt any One of the following :

(i) Find the Boolean function representing the following circuit :
Also find an equivalent circuit.



(2)

(ii) Construct the input-output table for the following Boolean function: (2)

$f(x_1, x_2) = x_1' \cdot x_2$

Q. 4 (A) (a) Attempt any One of the following :

(i) Find the 7th term of a sequence 3, 9, 20, 38, 65, ... using operators E and Δ. (3)

(ii) Evaluate : $\left(\frac{\Delta^3}{E^2}\right)(x^3)$ (3)

(b) Attempt any One of the following :

(i) Solve the differential equation : $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$, using $y = vx$. (3)

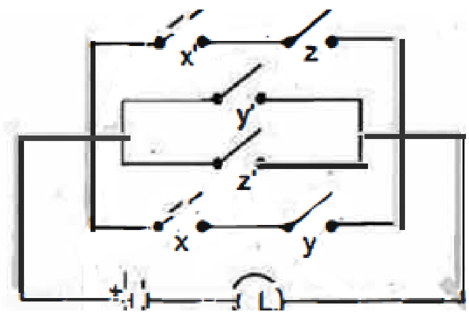
(ii) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple ? (3)

(B) Attempt any One of the following :

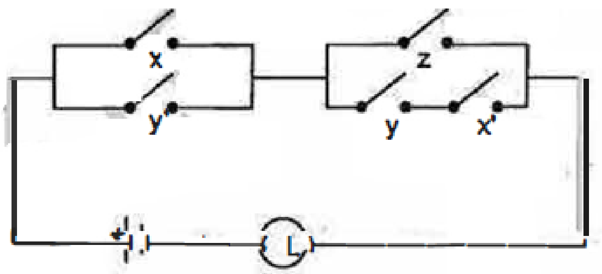
(i) If B is Boolean Algebra, then for any $x \in B$ prove that, $x + x = x$. (2)

(ii) Write the Boolean expression for the following circuits : (2)

(a)



(b)



Q. 5 (A) (a) Attempt any One of the following :

(i) If y is a differentiable function of u and u is a differential function of x, then prove that, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{dy}{dx}$. (3)

(ii) Prove that every differentiable function is continuous. (3)

(b) Attempt any One of the following:

(i) If u and v are differentiable functions of x, then prove that :

$$\int uv\,dx = u.\int v\,dx - \int\left[\frac{du}{dx} \cdot \int v dx\right]dx$$ (3)

(ii) Prove that :

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,fx \qquad \text{if } f(x) \text{ is even.}$$

$$= 0, \qquad \text{if } f(x) \text{ is odd.}$$ (3)

(B) Attempt any One of the following :

(i) Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ (2)

(ii) If $y = \frac{(\tan x)^x}{1 + x^2}$, find $\frac{dy}{dx}$. (2)