MATHS QUESTION PAPER

Time : 2 Hrs. **Max. Marks : 40**

Q. 1 (A) Attempt any TWO of the following : **[8]**

(i) State the truth values of the following prepositions. (3)

(a) The smallest prime number is 1.

(b) The square of an odd integer is odd.

(c) A quadratic equation cannot have more than two roots.

(ii) By constructing a truth table, verify whether the following statement pattern is a tautology, a contradiction or a contingency :

$[p \wedge (p \dashrightarrow \sim q)] \rightarrow q$ (3)

(iii) Construct the switching circuit for the following logical statement :

$(p \vee \sim q) \vee (q \wedge r)$ (3)

(B) Attempt any ONE of the following :

(i) A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. 4 per gram and Food F_2 costs Rs. 5 per gram. One gram of food F_1 contains minimum 3 units of vitamin A and 4 units of minerals. One gram of food F_2 contains minimum 6 units of vitamin A and 3 units of minerals Formulate this as L. P. P. to minimize the cost of diet. (2)

(ii) Draw a graph of the following inequalities :

$2x + 2y \geq 12, 5x + y \geq 10, x + 4y \geq 12, x \geq 0, y \geq 0.$

State only the vertices of the feasible region. (2)

Q. 2 (A) Attempt any TWO of the following : **[8]**

(i) If $\bar{a}, \bar{b}, \bar{c}$ are three non-zero, non-coplanar vectors, then prove that any vector $\bar{r}$ in space can be uniquely expressed as a linear combination of the vectors

$\bar{a}, \bar{b}, \bar{c}$ as $\bar{r} = x\bar{a} + y\bar{b} + z\bar{c}$, where x, y z are all non-zero scalars. (3)

(ii) By Vector method, prove that "medians of a triangle are concurrent". (3)

(iii) Show that points A, B, C, D are coplanar, where A = (2, 3, 5), B = (1, 1, 8), C = (5, 4, 1), D = (2, 2, 6). (3)

(B) Attempt any ONE of the following :

(i) If $\bar{a} = \bar{i} + 2\bar{j}, \bar{b} = 3\bar{i} + \bar{k}, \bar{c} = \bar{j} - \bar{k}$, find $\bar{a} \cdot (\bar{b} \times \bar{c})$. (2)

(ii) ABCDE is a pentagon, show that $\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} = 2\overrightarrow{AC}$. (2)

Q. 3 (A) (a) Attempt any ONE of the following : **[8]**

(i) Find the inverse of the following matrix = $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$, by using Adjoint method. (3)

(ii) Solve the following equations by Method of Reduction :

$x - y + z = 9;\ 2x + 5y + 7z = 52;\ 2x + y - z = 0$ (3)

(b) Attempt any ONE of the following :

(i) Show that every homogeneous equation of second degree in x and y represents a pair of straight lines passing through the origin. (3)

(ii) Find the condition that the line $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$. (3)

(B) Attempt any ONE of the following :

(i) Find the combined equation of the pair of lines through the origin such that one of them is parallel to $3x - y = 7$ and other is perpendicular to $2x + y = 8$. (2)

(ii) Find the equation of the circle having $(-1, 2)$ and $(3, -4)$ as the end points of diameter. (2)

Q. 4 (A) (a) Attempt any ONE of the following : [8]

(i) If the angle between the lines $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$, then prove that $100(h^2 - ab) = (a + b)^2$. (3)

(ii) A circle cuts off an intercept of 6 units from the line $3x - 4y - 2 = 0$. If the centre of the circle is $(-2, 3)$, find the equation of the circle. (3)

(b) Attempt any ONE of the following :

(i) Two cards are drawn at random from a well shuffled pack of 52 cards. Find the probability that the cards drawn contain one heart card and the other spade card. (3)

(ii) If A and B are any two events of a sample space S, then prove that :

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$. (3)

(B) Attempt any ONE of the following :

(i) Find the equation of the normal to the hyperbola $x^2 - 4y^2 = 36$ at point $(10, 4)$. (2)

(ii) Find the eccentricity and length of the latus rectum of the ellipse $3x^2 + 4y^2 = 1$. (2)

Q. 5 (A) (a) Attempt any ONE of the following : [8]

(i) Find the equation of the ellipse in the standard form whose distance between foci is 6 and eccentricity is $\frac{3}{5}$. (3)

(ii) Show that two tangents drawn from the point $(-6, 9)$ to the parabola $y^2 = 24x$ are at right angles. (3)

(b) Attempt any ONE of the following :

(i) Find the Vector equation and Cartesian equation of the line passing through two points $(1, -2, 1)$ and $(0, -2, 3)$. (3)

(ii) Find the angle between the planes $\bar{r}\,(2\bar{i} - \bar{j} + \bar{k}) = 6$ and $\bar{r}\,(\bar{i} + \bar{j} + 2\bar{k}) = 7$. (3)

(B) Attempt any ONE of the following :

(i) If e_1 and e_2 are the eccentricities of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ respectively, then prove that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$. (2)

(ii) Find the Carteian co-ordinate of the point on the parabola $y^2 = 8x$ whose parameter is 2. (2)