Board Question Paper: March 2013
Mathematics and Statistics

Time: 3 Hours   **Total Marks: 80**

Note:
i. All questions are compulsory
ii. Figures to the right indicate full marks.
iii. Solution of L.P.P. should be written on graph paper only.
iv. Answers to both the sections should be written in the same answer book.
v. Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following: **(6)[12]**

     i. The principal solution of the equation $\cot x = -\sqrt{3}$ is

       (A) $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{3}$

       (C) $\dfrac{5\pi}{6}$ (D) $-\dfrac{5\pi}{6}$

     ii. If the vectors $-3\hat{i}+4\hat{j}-2\hat{k},\ \hat{i}+2\hat{k},\ \hat{i}-p\hat{j}$ are coplanar, then the value of p is
       (A) -2 (B) 1
       (C) -1 (D) 2

     *iii. If the line $y = x + k$ touches the hyperbola $9x^2 - 16y^2 = 144$, then k = _____
       (A) 7   (B) -7
       (C) $\pm\sqrt{7}$ (D) $\pm\sqrt{19}$

(B) Attempt any THREE of the following: **(6)**
i. Write down the following statements in symbolic form:
     a. A triangle is equilateral if and only if it is equiangular.
     b. Price increases and demand falls.

ii. If A = $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, then find A^{-1} by adjoint method.

iii. Find the separate equations of the lines represented by the equation $3x^2 - 10xy - 8y^2 = 0$.

*iv. Find the equation of the director circle of a circle $x^2 + y^2 = 100$.

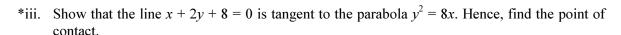v. Find the general solution of the equation $4\cos^2 x = 1$.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

     i. Without using truth table show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

     ii. If θ is the measure of acute angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left| \dfrac{2\sqrt{h^2 - ab}}{a+b} \right|$, $a + b \neq 0$.

*iii. Show that the line $x + 2y + 8 = 0$ is tangent to the parabola $y^2 = 8x$. Hence, find the point of contact.

(B) Attempt any TWO of the following: (8)

i. The sum of three numbers is 9. If we multiply third number by 3 and add to the second number, we get 16. By adding the first and the third number and then subtracting twice the second number from this sum, we get 6. Use this information and find the system of linear equations. Hence, find the three numbers using matrices.

ii. Find the general solution of $\cos x + \sin x = 1$.

iii. If $\bar{a}$ and $\bar{b}$ are any two non-zero and non-collinear vectors, then prove that any vector $\bar{r}$ coplanar with $\bar{a}$ and $\bar{b}$ can be uniquely expressed as $\bar{r} = t_1\bar{a} + t_2\bar{b}$, where t_1 and t_2 are scalars.

Q.3. (A) Attempt any TWO of the following: (6)[14]

i. Using truth table examine whether the following statement pattern is tautology, contradiction or contingency.
$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

ii. Find k, if the length of the tangent segment from $(8, -3)$ to the circle $x^2 + y^2 - 2x + ky - 23 = 0$ is $\sqrt{10}$ units.

iii. Show that the lines given by
$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1} \text{ and } \frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4} \text{ intersect.}$$
Also find the co-ordinates of the point of intersection.

(B) Attempt any TWO of the following: (8)

*i. Find the equation of the locus of the point of intersection of two tangents drawn to the hyperbola $\frac{x^2}{7} - \frac{y^2}{5} = 1$ such that the sum of the cubes of their slopes is 8.

ii. Solve the following L.P.P. graphically:
Maximize : $Z = 10x + 25y$
Subject to: $x \leq 3, y \leq 3, x + y \leq 5, x \geq 0, y \geq 0$

iii. Find the equations of the planes parallel to the plane $x + 2y + 2z + 8 = 0$ which are at the distance of 2 units from the point $(1, 1, 2)$.

Section − II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following:
(6)[12]

i. Function $f(x) = x^2 - 3x + 4$ has minimum value at $x =$ _____

   (A)   0   (B) $-\frac{3}{2}$

   (C)   1   (D) $\frac{3}{2}$

ii. $\int \frac{1}{x} \cdot \log x \, dx =$ _____

   (A) $\log(\log x) + c$ (B) $\frac{1}{2}(\log x)^2 + c$

   (C) $2 \log x + c$ (D) $\log x + c$

iii. Order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\frac{d^2y}{dx^2}$$ are respectively –

(A) 2, 3   (B) 3, 2
(C) 7, 2   (D) 3, 7

(B) **Attempt any THREE of the following:**   **(6)**

i. If $x = at^2$, $y = 2at$, then find $\frac{dy}{dx}$.

ii. Find the approximate value of $\sqrt{8.95}$.

iii. Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 3$.

*iv. For the bivariate data $r = 0.3$, $\text{cov}(X, Y) = 18$, $\sigma_x = 3$, find σ_y.

*v. A triangle bounded by the lines $y = 0$, $y = x$ and $x = 4$ is revolved about the X-axis. Find the volume of the solid of revolution.

Q.5. (A) **Attempt any TWO of the following:**   **(6)[14]**
i. A function f(x) is defined as
$$f(x) = x + a \quad , \qquad x < 0$$
$$= x \qquad , \quad 0 \le x < 1$$
$$= b - x \quad , \qquad x \ge 1$$
is continuous in its domain. Find a + b.

ii. If $x = a\left(t - \frac{1}{t}\right)$, $y = a\left(t + \frac{1}{t}\right)$, then show that $\frac{dy}{dx} = \frac{x}{y}$.

iii. Evaluate : $\displaystyle\int \frac{1}{3 + 5\cos x} dx$

(B) **Attempt any TWO of the following:**   **(8)**
i. An insurance agent insures lives of 5 men, all of the same age and in good health. The probability that a man of this age will survive the next 30 years is known to be $\frac{2}{3}$. Find the probability that in the next 30 years at most 3 men will survive.

ii. The surface area of a spherical balloon is increasing at the rate of 2 cm²/sec. At what rate is the volume of the balloon is increasing when the radius of the balloon is 6 cm?

iii. The slope of the tangent to the curve at any point is equal to $y + 2x$. Find the equation of the curve passing through the origin.

Q.6. (A) **Attempt any TWO of the following:**   **(6)[14]**
i. If u and v are two functions of x, then prove that
$$\int uv \, dx = u\int v \, dx - \int\left[\frac{du}{dx}\int v \, dx\right] dx$$

ii. The time (in minutes) for a lab assistant to prepare the equipment for a certain experiment is a random variable X taking values between 25 and 35 minutes with p. d. f.
$$f(x) = \frac{1}{10}, \ 25 \le x \le 35 \ = 0, \text{ otherwise.}$$
What is the probability that preparation time exceeds 33 minutes? Also find the c. d. f. of X.

iii. The probability that a certain kind of component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 tested components survive.

(B) **Attempt any TWO of the following:** **(8)**

i. If $ax^2 + 2hxy + by^2 = 0$, show that $\dfrac{d^2y}{dx^2} = 0$.

ii. Find the area of the region common to the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$.

*iii. For 10 pairs of observations on two variables X and Y, the following data are available:

$$\sum(x-2) = 30,\ \sum(y-5) = 40,\ \sum(x-2)^2 = 900,$$

$$\sum(y-5)^2 = 800,\ \sum(x-2)(y-5) = 480.$$

Find the correlation coefficient between X and Y.