Board Question Paper: October 2013
Mathematics and Statistics

Time: 3 Hours **Total Marks: 80**

Note:
i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Solution of L.P.P. should be written on graph paper only.
iv. Answers to both the sections should be written in the same answer book.
v. Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following: **(6)[12]**

i. If A = {2, 3, 4, 5, 6}, then which of the following is not true?

(A) $\exists\, x \in A$ such that $x + 3 = 8$ (B) $\exists\, x \in A$ such that $x + 2 < 5$

(C) $\exists\, x \in A$ such that $x + 2 < 9$ (D) $\forall\, x \in A$ such that $x + 6 \geq 9$

ii. If $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$, then the value of k is

(A) $\frac{1}{2}$ (B) $\frac{11}{2}$

(C) $\frac{5}{2}$ (D) $\frac{-11}{2}$

iii. If a line is inclined at 60° and 30° with the X and Y-axes respectively, then the angle which it makes with Z-axis is

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

(B) Attempt any THREE of the following: **(6)**

i. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and AX = I, then find X by using elementary transformations.

ii. With usual notations, in ΔABC, prove that $a(b \cos C - c \cos B) = b^2 - c^2$.

*iii. Show that the equation of a tangent to the circle $x^2 + y^2 = a^2$ at the point $P(x_1, y_1)$ on it is $xx_1 + yy_1 = a^2$.

*iv. Find k, if the line $2x - 3y + k = 0$ touches the ellipse $5x^2 + 9y^2 = 45$.

v. Find the co-ordinates of the point, which divides the line segment joining the points A(2, – 6, 8) and B(– 1, 3, – 4) externally in the ratio 1 : 3.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. Using truth table, prove that $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

ii. Find the values of p and q, if the following equation represents a pair of perpendicular lines: $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$.

*iii. Find the equations of tangents to the parabola $y^2 = 12x$ from the point (2, 5).

(B) Attempt any TWO of the following: **(8)**

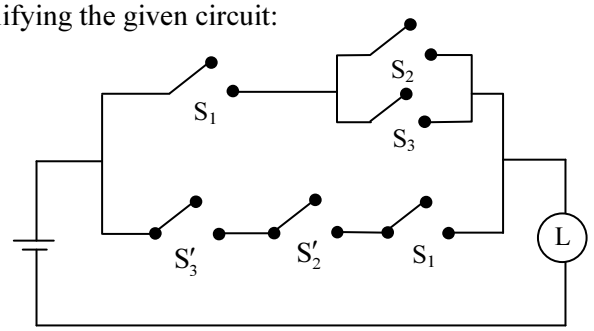
i. The cost of 2 books, 6 notebooks and 3 pens is ` 40. The cost of 3 books, 4 notebooks and 2 pens is ` 35, while the cost of 5 books, 7 notebooks and 4 pens is ` 61. Using this information and matrix method, find the cost of 1 book, 1 notebook and 1 pen separately.

ii. Prove that $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \cos^{-1}\left(-\frac{1}{2}\right)$.

*iii. Show that the product of lengths of perpendicular segments drawn from the foci to any tangent to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ is equal to 16.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Construct the new switching circuit for the following circuit with only one switch by simplifying the given circuit:



*ii. Find the locus of a point, the tangents from which to the circle $x^2 + y^2 = a^2$ are mutually perpendicular.

iii. Find the shortest distance between the lines

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

(B) Attempt any TWO of the following: **(8)**

i. Find the angle between the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z+2}{4}$ and the plane $2x + y - 3z + 4 = 0$.

ii. Solve the following L. P. P. graphically:

Minimize $Z = 6x + 2y$

Subject to

$5x + 9y \leq 90$

$x + y \geq 4$

$y \leq 8$

$x \geq 0, y \geq 0$

iii. Find the volume of a tetrahedron whose vertices are

A(−1, 2, 3), B(3, −2, 1), C(2, 1, 3) and D(−1, −2, 4).

SECTION - II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following: (6)[12]

i. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ = _____

(A) $\frac{1+x}{1+\log x}$ (B) $\frac{\log x}{(1+\log x)^2}$ (C) $\frac{1-\log x}{1+\log x}$ (D) $\frac{1-x}{1+\log x}$

ii. $\int \frac{1}{1+\cos x}\,dx$ = _____

(A) $\tan\left(\frac{x}{2}\right)+c$ (B) $2\tan\left(\frac{x}{2}\right)+c$

(C) $-\cot\left(\frac{x}{2}\right)+c$ (D) $-2\cot\left(\frac{x}{2}\right)+c$

iii. If X ~ B (n, p) and E(X) = 12, Var(X) = 4, then the value of n is _____

(A) 3 (B) 48 (C) 18 (D) 36

(B) Attempt any THREE of the following: (6)

i. Find the equation of tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3).

ii. Evaluate: $\int \frac{1}{x(x-1)}dx$

iii. Solve the differential equation $y - x\frac{dy}{dx} = 0$.

*iv. In a bivariate data, n = 10, $\bar{x} = 25$, $\bar{y} = 30$ and $\sum xy = 7900$.
Find cov(X, Y).

*v. A random variable X ~ N (0, 1). Find P(X > 0) and P(X < 0).

Q.5. (A) Attempt any TWO of the following: (6)[14]

i. Examine the function for maximum and minimum $f(x) = x^3 - 9x^2 + 24x$.

ii. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$, where $\frac{dy}{dx} \neq 0$.

iii. The probability distribution of X, the number of defects per 10 metres of a fabric is given by

x	0	1	2	3	4
P(X = x)	0.45	0.35	0.15	0.03	0.02

Find the variance of X.

(B) Attempt any TWO of the following: (8)

i. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

ii. Solve the differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$.

iii. Find the area of the region bounded by the curves $y^2 = 4x$ and $4x^2 + 4y^2 = 9$ with $x \geq 0$.

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Find the approximate value of $\tan^{-1}(1.001)$.

ii. Examine continuity of the function f(x) at $x = 0$, where

$$f(x) = \frac{10^x + 7^x - 14^x - 5^x}{1 - \cos 4x} \quad , \text{for } x \neq 0$$

$$= \frac{10}{7} \quad , \text{for } x = 0$$

iii. The probability that a person who undergoes a kidney operation will be recovered is 0.5. Find the probability that of the 6 patients who undergo similar operations:
(a) none will recover
(b) half of them will recover.

(B) Attempt any TWO of the following: **(8)**

i. Prove that:

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left| x + \sqrt{x^2 + a^2} \right| + c$$

*ii. Find the volume of the solid generated, when the area between ellipse $4x^2 + 9y^2 = 36$ and the chord AB, with A ≡ (3, 0), B ≡ (0, 2), is revolved about X-axis.

*iii. Find Karl Pearson's coefficient of correlation between the variables X and Y for the following data:

X	11	7	9	5	8	6	10
Y	10	8	6	5	9	7	11