BOARD QUESTION PAPER : OCTOBER 2015
MATHEMATICS AND STATISTICS

Time: 3 Hours **Total Marks: 80**

Note:

i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Answer to every new question must be written on a new page.
v. Answers to both sections should be written in the same answer book.
vi. Use of logarithmic table is allowed.

SECTION – I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If $p \wedge q = F$, $p \rightarrow q = F$, then the truth value of p and q is :

(A) T, T (B) T, F
(C) F, T (D) F, F

ii. If $A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ and $|A| = 3$, then (adj. A) = ________

(A) $\frac{1}{9}\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -5 & -2 \\ -2 & 4 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & -4 & 2 \\ 2 & 5 & -4 \\ 1 & -2 & 1 \end{bmatrix}$

iii. The slopes of the lines given by $12x^2 + bxy - y^2 = 0$ differ by 7. Then the value of b is :

(A) 2 (B) ± 2
(C) ± 1 (D) 1

(B) Attempt any THREE of the following: **(6)**

i. In a Δ ABC, with usual notations prove that:

$$\frac{a - b\cos C}{b - a\cos C} = \frac{\cos B}{\cos A}$$

ii. Find 'k', if the equation $kxy + 10x + 6y + 4 = 0$ represents a pair of straight lines.

iii. If A, B, C, D are four non-collinear points in the plane such that $\overline{AD} + \overline{BD} + \overline{CD} = \overline{O}$, then prove that point D is the centroid of the ΔABC.

iv. Find the direction cosines of the line

$$\frac{x+2}{2} = \frac{2y-5}{3};\ z = -1.$$

v. Show that the points (1, 1, 1) and (–3, 0, 1) are equidistant from the plane $\hat{r}.\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$.

Q.2. (A) Attempt any TWO of the following: (6)[14]

i. Using truth table prove that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$.

ii. Prove that a homogeneous equation of second degree, $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines passing through the origin, if $h^2 - ab \geq 0$.

iii. Prove that the volume of a parallelopiped with coterminal edges as $\bar{a}, \bar{b}, \bar{c}$ is $[\bar{a}, \bar{b}, \bar{c}]$. Hence find the volume of the parallelopiped with coterminal edges $\bar{i}+\bar{j}, \bar{j}+\bar{k}$ and $\bar{k}+\bar{i}$.

(B) Attempt any TWO of the following: (8)

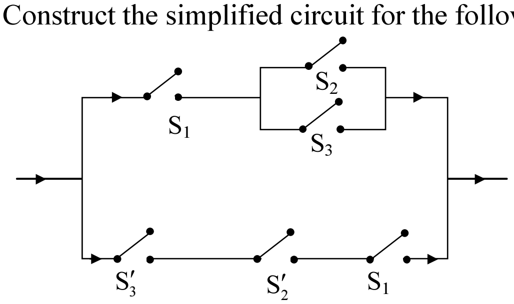
i. Find the inverse of the matrix, $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by using column transformations.

ii. In ΔABC, prove that : $\tan\frac{(A-B)}{2} = \left(\frac{a-b}{a+b}\right).\cot\frac{C}{2}$.

iii. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$; and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Find the equation of the plane containing them.

Q.3. (A) Attempt any TWO of the following: (6)[14]

i. Construct the simplified circuit for the following circuit:



ii. Express $-\hat{i}-3\hat{j}+4\hat{k}$ as a linear combination of vectors $2\hat{i}+\hat{j}-4\hat{k}, 2\hat{i}-\hat{j}+3\hat{k}$ and $3\hat{i}+\hat{j}-2\hat{k}$.

iii. Find the length of the perpendicular from the point (3, 2, 1) to the line $\frac{x-7}{-2} = \frac{y-7}{2} = \frac{z-6}{3}$.

(B) Attempt any TWO of the following: (8)

i. Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

ii. Minimize : $Z = 6x + 4y$

Subject to the conditions:

$3x + 2y \geq 12$,

$x + y \geq 5$,

$0 \leq x \leq 4$,

$0 \leq y \leq 4$

iii. If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \cot^{-1}\left(\frac{x+2}{x+1}\right) = \frac{\pi}{4}$; find x.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If $y = \sec^{-1}\left(\frac{\sqrt{x}-1}{x+\sqrt{x}}\right) + \sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)$, then $\frac{dy}{dx} = ...$

(A) x (B) $\frac{1}{x}$

(C) 1 (D) 0

ii. If $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$, then the value of I is:

(A) 0 (B) π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

iii. The solution of the differential equation $\frac{dy}{dx} = \sec x - y \tan x$ is:

(A) $y \sec x = \tan x + c$ (B) $y \sec x + \tan x = c$

(C) $\sec x = y \tan x + c$ (D) $\sec x + y \tan x = c$

(B) Attempt any THREE of the following: **(6)**

i. Evaluate: $\int \frac{1}{x \log x \log(\log x)} dx$

ii. Find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

iii. Find k, such that the function

$$P(x) = \begin{cases} k\binom{4}{x}; x = 0,1,2,3,4. k > 0 \\ 0 \quad ;\text{otherwise}; \end{cases}$$

Is a probability mass function (p.m.f.).

iv. Given is X ~ B (n, p). If E(X) = 6, and Var(X) = 4.2, find the value of n.

v. Solve the differential equation $y - x\frac{dy}{dx} = 0$

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. Discuss the continuity of the function

$$f(x) = \frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}, \text{ for } x \neq \frac{\pi}{2}$$

$$= 3, \quad \text{for } x = \frac{\pi}{2},$$

at $x = \frac{\pi}{2}$

ii. If $f'(x) = k(\cos x - \sin x)$, $f'(0) = 3$ and $f\left(\frac{\pi}{2}\right) = 15$,

find $f'(x)$.

iii. Differentiate $\cos^{-1}\left(\frac{3\cos x - 2\sin x}{\sqrt{13}}\right)$ w. r. t. x.

(B) Attempt any TWO of the following: **(8)**

i. Show that: $\int \frac{1}{x^2\sqrt{a^2+x^2}}\,dx = \frac{-1}{a^2}\frac{\sqrt{a^2+x^2}}{x}+c$

ii. A rectangle has area 50 cm^2. Find its dimensions when its perimeter is the least.

iii. Prove that : $\int_{-a}^{a} f(x)\,dx = 2.\int_{0}^{a} f(x)\,dx$, if $f(x)$ is an even function.

$= 0$, if $f(x)$ is an odd function.

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. If $y = f(u)$ is a differential function of u and $u = g(x)$ is a differential function of x, then prove that $y = f[g(x)]$ is a differential function of x and $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$

ii. Each of the total five questions in a multiple choice examination has four choices, only one of which is correct. A student is attempting to guess the answer. The random variable x is the number of questions answered correctly. What is the probability that the student will give atleast one correct answer?

iii. If $f(x) = x^2 + a$, for $x \geq 0$

$= 2\sqrt{x^2+1} + b$, for $x < 0$ and $f\left(\frac{1}{2}\right) = 2$,

is continuous at $x = 0$, find a and b.

(B) Attempt any TWO of the following: **(8)**

i. Find the approximate value of cos (89°, 30′). [Given is: 1° = 0·0175°C]

ii. Solve the differential equation:

$x + y\frac{dy}{dx} = \sec(x^2 + y^2)$. Also find the particular solution if $x = y = 0$.

iii. Find the expected value, variance and standard deviation of random variable X whose probability mass function (p.m.f.) is given below:

$X = x$	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$