

WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सैनियर स्कूल सर्टिफिकेट परीक्षा (फका बाइसबी)
परीक्षार्थी प्रवेश-पत्र के अनुसार भरे

विषय Subject: **MATHEMATICS**

विषय कोड Subject Code: **041**

परीक्षा का दिन (in Hindi)

Day & Date of the Examination: **MONDAY 14.03.2016**

उत्तर देने का माध्यम

Medium of answering the paper: **ENGLISH**

प्रश्न पत्र के ऊपर लिखें

कोड का कोड

Write code No. as written on

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NIL

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हाँ / No

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No

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Each letter to reflect in arabic and one has to left blank between each part of the

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Section - A

$$\vec{r}_0 \cdot \hat{n} = d$$

1. $\hat{n}$ - Unit vector perpendicular to the plane

d - Distance of the plane from origin.

$$\vec{r}_0 \cdot (2\hat{i} + 3\hat{j} + 6\hat{k}) = 4$$

$$\vec{r}_0 \cdot (2\hat{i} + 3\hat{j} + 6\hat{k}) = \frac{4}{\sqrt{2^2 + 3^2 + 6^2}} = \frac{4}{\sqrt{49}} = \frac{4}{7}$$

$$\Rightarrow \vec{r}_0 \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k} \right) = \frac{4}{7} \quad (\text{Distance of plane from origin} = \frac{4}{7})$$

$$\vec{r}_0 \cdot (6\hat{i} + 9\hat{j} + 18\hat{k}) = -30$$

$$\Rightarrow \vec{r}_0 \cdot (6\hat{i} + 9\hat{j} + 18\hat{k}) = \frac{-30}{\sqrt{6^2 + 9^2 + 18^2}} = \frac{-30}{\sqrt{36 + 81 + 324}} = \frac{-30}{\sqrt{441}} = \frac{-30}{21} = \frac{-10}{7}$$

(Distance of plane from origin = $\frac{10}{7}$ in the direction opposite to the unit vector)

$$\therefore \text{Distance between the planes} = \frac{4}{7} - \left(\frac{-10}{7} \right) = \frac{14}{7} = 2 \text{ units}$$

2. $\vec{a} - \vec{b}$ is a unit vector

$$|\vec{a} - \vec{b}| = 1$$

$$|\vec{a} - \vec{b}|^2 = 1$$

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 1$$

$$1 + 2 - 2\vec{a} \cdot \vec{b} = 1 \Rightarrow \vec{a} \cdot \vec{b} = \frac{1}{2}$$

($|\vec{a}| = |\vec{b}| = 1$) (θ : Angle between vectors $\vec{a}$ & $\vec{b}$)

$$\therefore |\vec{a}| |\vec{b}| \cos \theta = \frac{1}{2} \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 45^\circ \quad (\text{Angle between } \vec{a} \text{ \& } \vec{b} \text{ is } 45^\circ)$$

$$3. \quad |\vec{a}| = \frac{1}{2} \quad |\vec{b}| = \frac{4}{\sqrt{3}} \quad |\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}} \quad |\vec{a} \cdot \vec{b}| = ?$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta \quad (\theta: \text{Angle between vectors } \vec{a} \text{ \& } \vec{b})$$

$$\frac{1}{\sqrt{3}} = \frac{1}{2} \times \frac{4}{\sqrt{3}} \sin \theta$$

$$\sin \theta = \frac{1}{2} \quad \theta = 30^\circ$$

$$|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos \theta$$

$$= \frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$$

$$|\vec{a} \cdot \vec{b}| = 1$$

$$4. \quad A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$$

$$kA = \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix}$$

$$\text{But given } kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

Equating individual terms,

$$2k = -8$$

$$k = -4$$

$$3k = 4a$$

$$3 \times (-4) = 4a$$

$$-12 = 4a$$

$$a = -3$$

$$b = -k = 4$$

$$|AB| = |A||B|$$

(Provided A & B are square matrices)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 1 \times (-1) - 2 \times 3 = -7$$

$$|B| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} = 1 \times (-2) - 3 \times (-4) = 10$$

$$\therefore |AB| = -7 \times 10 = -70$$

$$|A| = 5$$

$$|AA^T| = |A||A^T|$$

(As A is a square matrix)

$$= |A||A|$$

$$(As |A| = |A^T|)$$

$$= |A|^2 = 25$$

Section-B

7. To prove: $2 \sin^{-1}\left(\frac{2}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

Proof: Let $\sin^{-1}\left(\frac{2}{5}\right) = \theta$

$\therefore \sin \theta = \frac{2}{5}$

$\therefore \tan \theta = \frac{3}{4}$ (From triangle ①)



Triangle ①

$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right)$

Now $2 \tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{1 - \frac{9}{16}}\right) = \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) = \tan^{-1}\left(\frac{3 \times 8}{7}\right) = \tan^{-1}\left(\frac{24}{7}\right)$

$\therefore 2 \sin^{-1}\left(\frac{2}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24 \times 17}{7 \times 31}}\right)$

$= \tan^{-1}\left(\frac{24 \times 31 - 17 \times 7}{7 \times 31 + 24 \times 17}\right) = \tan^{-1}\left(\frac{625}{625}\right) = \tan^{-1}(1) = \frac{\pi}{4}$ Hence Proved

$$e. \text{ Let } I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx \quad \text{--- (1)}$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin^2(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)} dx$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx \quad \text{--- (2)}$$

Adding (1) & (2)

$$2I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x} \quad \left\{ \cos(A-B) = \cos A \cos B + \sin A \sin B \right\}$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x} = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x + \sin x + \frac{1}{\sqrt{2}}}$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos(x - \frac{\pi}{4})}$$

Put $x - \frac{\pi}{4} = t$
 $dx = dt$

For $x=0$, $t = -\frac{\pi}{4}$, for $x = \frac{\pi}{2}$, $t = \frac{\pi}{4}$

$$\begin{aligned}
 I &= \frac{1}{2\sqrt{2}} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec t \, dt \\
 &= \frac{1}{2\sqrt{2}} \log \left| \sec t + \tan t \right| \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\
 &= \frac{1}{2\sqrt{2}} \left(\log \left| \sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right| - \log \left| \sec \left(-\frac{\pi}{4} \right) + \tan \left(-\frac{\pi}{4} \right) \right| \right) \\
 &= \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}+1}{\sqrt{2}-1} \right| \\
 &= \frac{1}{2\sqrt{2}} \log \left| \frac{(\sqrt{2}+1)^2}{2-1} \right| = \frac{1}{\sqrt{2}} \log |\sqrt{2}+1|
 \end{aligned}$$

9. Let $I = \int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$

$$\begin{aligned}
 &= \int \log(\log x) \, dx + \int \frac{dx}{(\log x)^2} \\
 &= I_1 + I_2 + C \text{ (say)} \quad \{ C = \text{Arbitrary constant} \} \\
 I_1 &= \int \log(\log x) \, dx \quad \& \quad I_2 = \int \frac{dx}{(\log x)^2}
 \end{aligned}$$

Consider $I_1 = \int \log(\log x) \cdot x \cdot dx$

$$= x \log(\log x) - \int x \cdot \frac{1}{\log x} \cdot x \cdot \frac{1}{x} dx$$

{Applying Integration by parts}

$$= x \log(\log x) - \int \frac{dx}{\log x}$$

$$= x \log(\log x) - \left[\frac{x}{\log x} - \int \frac{x \cdot \frac{-1}{(\log x)^2} \cdot dx}{x} \right]$$

{Applying Integration by parts}

$$I_1 = x \log(\log x) - \frac{x}{\log x} - \int \frac{dx}{(\log x)^2}$$

But $\int \frac{dx}{(\log x)^2} = I_2$

$$\therefore I = I_1 + I_2 + C = x \log(\log x) - \frac{x}{\log x} + C$$

$$10. \text{ let } I = \int \frac{(1 - \sin x) dx}{\sin x (1 + \sin x)}$$

$$= \int \frac{(1 - \sin x)(1 - \sin x) dx}{\sin x (1 + \sin x)(1 - \sin x)}$$

Multiplying numerator & Denominator by $1 - \sin x$

$$= \int \frac{(1 - \sin x)^2 dx}{\sin x (1 - \sin^2 x)} = \int \frac{(1 + \sin^2 x - 2 \sin x) dx}{\sin x \cos^2 x}$$

$$= \int \frac{dx}{\sin x \cos^2 x} + \int \frac{\sin x dx}{\cos^2 x} - 2 \int \frac{dx}{\cos^2 x}$$

$$= \int \frac{\sin x dx}{(\cos^2 x) \cos^2 x} + \int \frac{\sin x dx}{\cos^2 x} - 2 \int \sec^2 x$$

" I₁ " I₂ (say)

$$I_2 = \int \frac{\sin x dx}{\cos^2 x}$$

Put $\cos x = t$
 $-\sin x dx = dt$

$$\therefore I_2 = \int \frac{-dt}{t^2} = \frac{1}{t} = \sec x + C$$

$$I_1 = \int \frac{\sin x \, dx}{(1 - \cos^2 x)(\cos^3 x)}$$

Put $\cos x = u$

$$-\sin x \, dx = du$$

$$I_1 = \int \frac{-du}{(1-u^2)u^3} = \int \frac{du}{(u^2-1)u^2}$$

$$= \int \frac{du}{u^2-1} - \int \frac{du}{u^2}$$

$$= \frac{1}{2} \log \left| \frac{u-1}{u+1} \right| + \frac{1}{u} + C = \int \frac{du}{u^2-1} = \int \frac{du}{(u+1)(u-1)}$$

$$I_1 = \frac{1}{2} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + \sec x + C = \frac{1}{2} \log \left| \frac{(u+1)-(u-1)}{(u+1)(u-1)} \right| du$$

$$= \frac{1}{2} \log \left| \frac{u-1}{u+1} \right|$$

$$\therefore I_1 = \int \frac{(1-\sin x) \, dx}{\sin x (1+\sin x)} = \frac{1}{2} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + 2 \sec x - 2 \tan x + k$$

k is an

arbitrary constant.

$$x = am^2$$

$$ay^2 = (am^2)^3$$

$$ay^2 = a^3 m^6$$

$$y^2 = a^2 m^6$$

$$\therefore y = \pm am^3$$

Considering $(x, y) = (am^2, am^3)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dm}}{\frac{dx}{dm}}$$

$$\frac{dy}{dm} = \frac{d(am^3)}{dm} = 3am^2$$

$$\frac{dx}{dm} = \frac{d(am^2)}{dm} = 2am$$

$$\therefore \frac{dy}{dx} = \frac{3am^2}{2am} = \frac{3m}{2} \quad \text{Slope of tangent at } (am^2, am^3)$$

$$\text{Slope of normal at } (am^2, am^3) = -\frac{1}{\frac{dy}{dx}} = -\frac{1}{\frac{3m}{2}} = -\frac{2}{3m}$$

Equation of normal to the curve at (am^2, am^3)

$$y - am^3 = -\frac{2}{3m} (x - am^2)$$

$$3my - 3am^4 = -2x + 2am^2$$

$3my + 2x = 3am^4 + 2am^2$ is the required equation.

$$f(x) = \begin{cases} k \sin\left(\frac{\pi}{2}(x+1)\right) & x \leq 0 \\ \frac{\tan x - \sin x}{x^2} & x > 0 \end{cases}$$

$f(x)$ is continuous at $x=0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\text{Now } \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^-} k \sin\left(\frac{\pi}{2}(x+1)\right) = k \sin \frac{\pi}{2} = k$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\tan x - \sin x}{x^2} = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \left(\frac{\sec x - 1}{x^2} \right)$$

$$= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} = \lim_{x \rightarrow 0^+} \frac{1 - \cos x}{\cos x \cdot x^2}$$

$$= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \times \lim_{x \rightarrow 0^+} \frac{2 \sin^2 \frac{x}{2}}{4x \left(\frac{x}{2}\right)^2}$$

$$\lim_{x \rightarrow 0^+} \cos x$$

$$\text{Now } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \& \quad \lim_{x \rightarrow 0} \left(\frac{\sin(\frac{x}{2})}{(\frac{x}{2})} \right)^2 = 1$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = \frac{1 \times \frac{1}{2}}{1} = \frac{1}{2}$$

$$\therefore h = \frac{1}{2}$$

{ As $f(x)$ is continuous at $x=0$ }

$$\text{Consider } \tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$$

$$= \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right)$$

$$\text{Put } \text{But } x^2 = \cos 2\theta \quad \therefore \theta = \frac{1}{2} \cos^{-1} x^2$$

$$\therefore \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right) = \tan^{-1} \left(\frac{1 - \frac{1 - \cos 2\theta}{1 + \cos 2\theta}}{1 + \frac{1 - \cos 2\theta}{1 + \cos 2\theta}} \right)$$

$$1 - \cos 2\theta = 2 \sin^2 \theta$$

$$1 + \cos 2\theta = 2 \cos^2 \theta$$

$$\therefore \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right) = \tan^{-1} \left(\frac{1 - \tan \theta}{1 + \tan \theta} \right) = \tan^{-1} \left(\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} \right)$$

$$= \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \theta \right) \right)$$

$$\frac{\pi - \theta}{4}$$

We want

$$\frac{d}{d(\cos^2 x)} \left(\tan^{-1} \left(\frac{\sqrt{1-x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right) \right) = \frac{d}{d\theta} \left(\frac{\pi - \theta}{4} \right) = \frac{d}{d\theta} \left(\frac{\pi - \theta}{4} \right)$$

$$= \frac{1}{4} \frac{d}{d\theta} \left(\frac{\pi - \theta}{4} \right) = \frac{1}{4} \frac{d}{d\theta} \left(\frac{\pi - \theta}{4} \right)$$

$$= \frac{1}{4} \frac{d}{d\theta} \left(\frac{\pi - \theta}{4} \right)$$

14. A plane which passes through $A(3, 2, 1)$, $B(4, 2, -2)$ & $C(6, 5, -1)$

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 4-3 & 2-2 & -2-1 \\ 6-3 & 5-2 & -1-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix}$$

$$0 = (x-3)(0 \times (-2) - (-3) \times 3) - (y-2)(1 \times (-2) - (-3) \times 3) + (z-1)(1 \times 3 - 3 \times 0)$$

$$9(x-3) - 7(y-2) + 3(z-1) = 0$$

$$9x - 7y + 3z = 27 - 14 + 3 = 16$$

$\therefore$ Plane passing through points A, B, C is $9x - 7y + 3z = 16$

Now A, B, C & $D(2, 5, 5)$ are coplanar.

$\therefore D$ lies on the plane of A, B, C

$$9 \times 2 - 7 \times 5 + 3 \times 5 = 16$$

$$9 \times 2 = 18$$

$$-7 \times 5 = -35$$

$$+3 \times 5 = 15$$

$$18 - 35 + 15 = 16$$

$$\therefore \lambda = 4$$

$$\vec{a} = \vec{b} + \vec{c}$$



Hence $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$

$$\vec{b} = 3\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{a} = \vec{b} + \vec{c}$$

$$p\hat{i} + q\hat{j} + r\hat{k} = 3\hat{i} + 3\hat{j} + 4\hat{k} + 3\hat{i} + \hat{j} - 2\hat{k}$$

$$p\hat{i} + q\hat{j} + r\hat{k} = (3+3)\hat{i} + 4\hat{j} + 2\hat{k}$$

Equating components,

$$p = 3+3$$

$$q = 4$$

$$r = 2$$

Area of triangle = $\frac{1}{2} |\vec{a} \times \vec{b}|$

But Area of triangle = $\frac{1}{2} |\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p & q & r \\ 3 & 3 & 4 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 10r - (3p+6)r + (0-p)r \end{vmatrix}$$

$$600 = 100 + (3p+6)^2 + (12-p)^2$$

$$500 = 4p^2 + 36 + 24p + 144 + p^2 - 24p \Rightarrow 5p^2 = 320$$

$$\frac{360}{40} = \frac{320}{80}$$

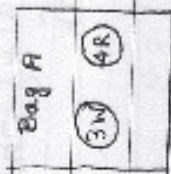
$$p^2 = 64$$

$$p = \pm 8$$

$$p = 8, \quad 5 = p - 3 = 5$$

$$p = -8, \quad 5 = p - 3 = -11$$

16



Let E_1 : Event that the balls are drawn from bag A

E_2 : Event that the balls are drawn from bag B

C: Event that 1 white ball & 1 red ball are drawn

$$P(E_2|C) = ?$$

$$\text{Now } P(E_1) = \frac{1}{2} = P(E_2)$$

$$P(C|E_1) = \frac{3}{7} \times \frac{2}{6} \times \frac{4}{5} \times \frac{3}{4} = \frac{4}{35}$$

$$P(C|E_2) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{6}{35}$$

(Multiplied by 3 because they can be chosen in any order)

By Bayes theorem,

$$P(E_2|C) =$$

$$\frac{P(E_2)P(C|E_2)}{P(E_1)P(C|E_1) + P(E_2)P(C|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{6}{35} \times 3}{\frac{1}{2} \times \frac{4}{35} \times 3 + \frac{1}{2} \times \frac{6}{35} \times 3} = \frac{6}{10} = \frac{3}{5}$$

$$= \frac{1}{2} \times \frac{6}{35} \times 3$$

$$\frac{1 \times 4 \times 3 + 1 \times 6 \times 3}{2 \times 35} = \frac{6}{10} = \frac{3}{5}$$

17.

Let the length of the plot be l .

Let the breadth of the plot be b .

Now $l \times b = A$ (Area of the plot)

$$(l-50)(b+50) = A$$

$$(l-10)(b-20) = A-5300$$

$$lb = 50b + 50l - 2000 = A$$

$$-b + l = 50 \quad (A=20)$$

$$lb - 10b = 20l + 200 = A - 5300$$

$$10b + 20l = 5500$$

$$b + 2l = 550$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$A X = B$$

$$A X = B$$

$$\therefore X = A^{-1} B$$

$$\text{When } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(ad-bc = 1 \times 1 - (-2) \times 1 = 3 \neq 0)$$

$\therefore A^{-1}$ exists

$$\text{Here } A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1+2} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 550 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 600 \\ 450 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$a = 200 \text{ m}$$

$$b = 150 \text{ m}$$

He wants to donate the plot to the school because he wants rural places to become developed & he is thereby showing his kind heartedness & his intention to help the society to develop by producing more literates. Children should have an opportunity to learn.

$$2ye^{\frac{x}{y}} dx + (y - 2xe^{\frac{x}{y}}) dy = 0$$

$$\frac{dy}{dx} = -\frac{y - 2xe^{\frac{x}{y}}}{2ye^{\frac{x}{y}}}$$

$$y - 2xe^{\frac{x}{y}}$$

$$\frac{dx}{dy} = \frac{y - 2xe^{\frac{x}{y}}}{-2ye^{\frac{x}{y}}} = f(x, y) \text{ (say)}$$

$$F(x, y) = \frac{y - 2\lambda x e^{\frac{\lambda x}{y}}}{-2\lambda y e^{\frac{\lambda x}{y}}} = \lambda^0 f(x, y)$$

The equation is a homogeneous differential equation

Put $x = vy$

$$\frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$\therefore v + y \frac{dv}{dy} = \frac{y - 2vy e^v}{-2ye^v} = -\frac{1}{2}e^{-v} + v \Rightarrow y \frac{dv}{dy} = -\frac{1}{2}e^{-v}$$

$$\therefore e^y dy = \frac{1}{2} \frac{dy}{y}$$

Integrating

$$e^y = -\frac{1}{2} \log y + C$$

$\therefore e^y = -\frac{1}{2} \log y + C$ is the required solution to the differential equation.

$$(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^3$$

$$\frac{dy}{dx} - \frac{y}{(x+1)} = e^{3x} (x+1)^2$$

This equation is of the form $\frac{dy}{dx} + Py = Q$ { P, Q = functions of x alone }

Linear differential equation.

$$\therefore \text{Integrating factor} = e^{\int P dx} = e^{-\int \frac{dx}{x+1}} = \frac{1}{x+1}$$

Solution to the differential equation:

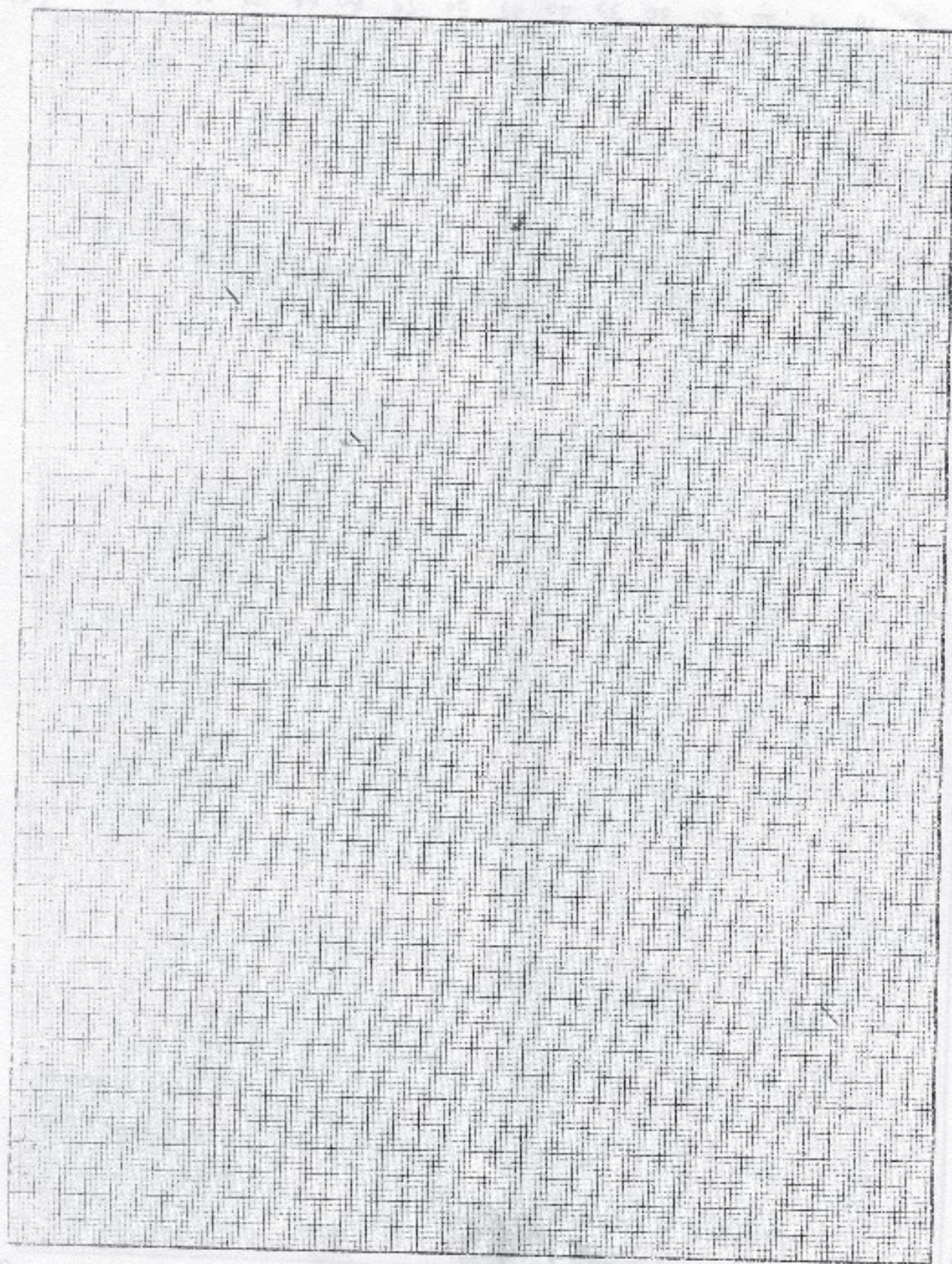
$$y \times I.F. = \int Q \times I.F. dx + C$$

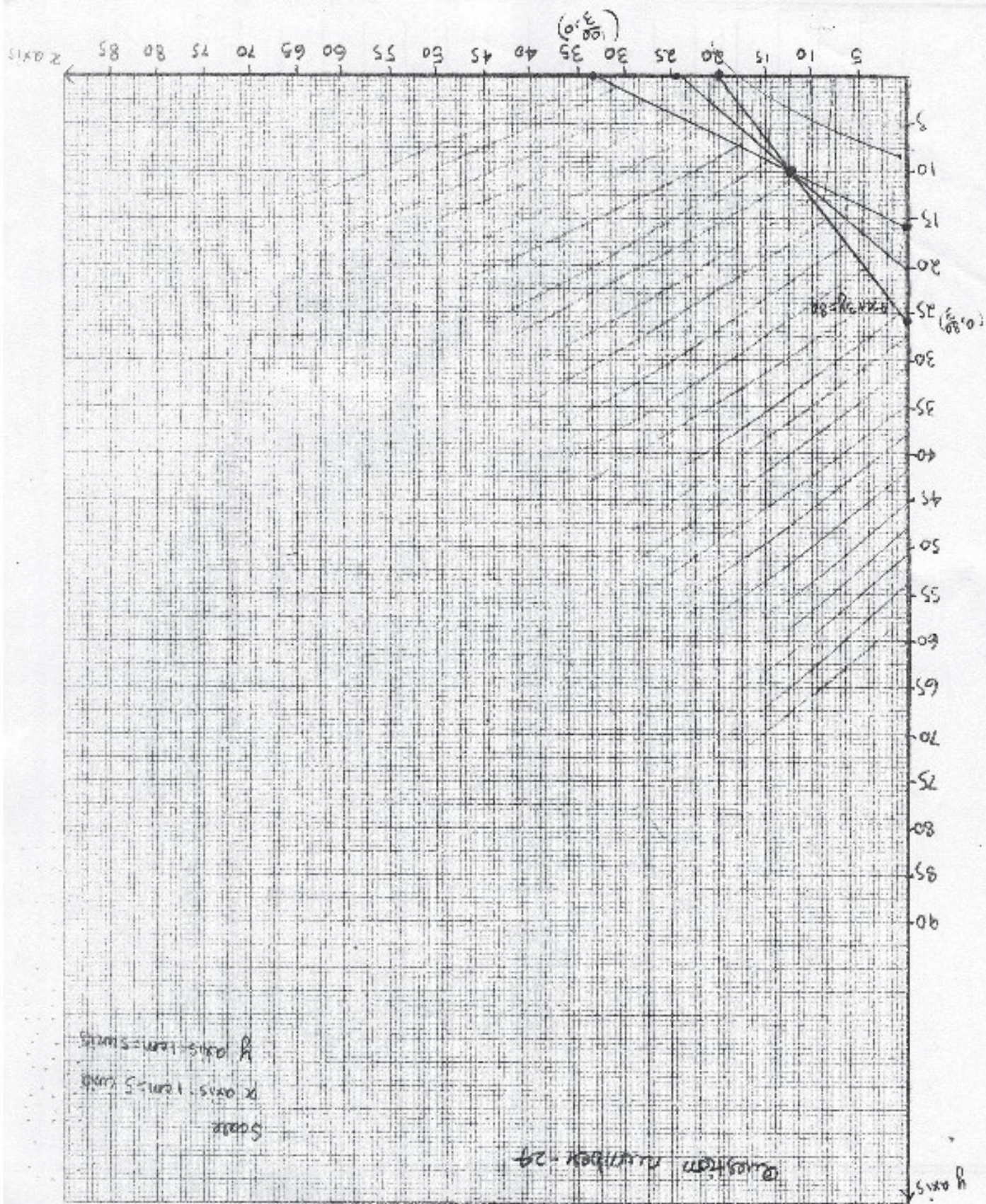
$$\therefore \frac{y}{x+1} = \int e^{3x} (x+1)^2 \frac{1}{(x+1)} dx + C$$

$$= \int x e^{3x} dx + \int e^{3x} dx + C = \frac{x e^{3x}}{3} - \frac{1}{9} \int e^{3x} dx + \int e^{3x} dx + C$$

$$\frac{y}{x+1} = \frac{x e^{3x}}{3} + \frac{2}{9} e^{3x} + C$$

is the solution to the required differential equation





Section-C

24 Let the number of units of Food F_1 Vitamin A = x

Let the number of units of Food F_2 Vitamin B = y

$$x \geq 80 \quad x \geq 0 \quad y \geq 0$$

$$y \geq 100 \quad y \geq 0$$

$$4x + 3y \geq 80$$

$$3x + 6y \geq 100$$

$$4x + 3y = 80$$

$$3x + 6y = 100$$

$$8x + 6y = 160$$

$$5x = 60$$

$$x = 12$$

$$y = \frac{32}{3}$$

Minimize Z

$$Z = 5x + 6y$$

$$(x, y)$$

$$(0, \frac{80}{3})$$

$$(\frac{100}{3}, 0)$$

$$(12, \frac{32}{3})$$

$$Z$$

$$Z = 160$$

$$Z = 166.67$$

$$Z = 124$$

Since the region is unbounded, we should check if any point other than $(12, \frac{32}{3})$ is in common to region shaded, & $5x + 6y \leq 124$.
 Clearly, from the graph, only $(12, \frac{32}{3})$ lies on both regions.
 $\therefore$ The Minimum cost of the diet = $\underline{\underline{₹. 124}}$
 with 12 units of Food F_1 & $\frac{32}{3}$ units of Food F_2

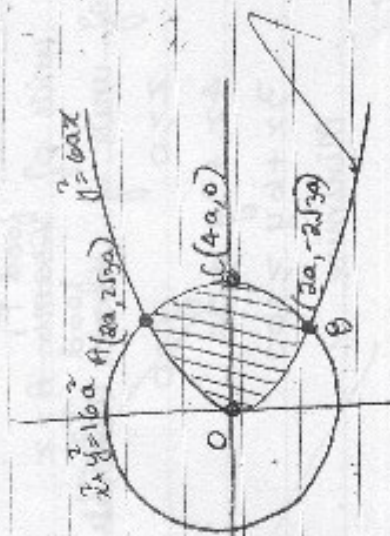
Solving the two curves

$$x^2 + 6ax = 16a^2$$

$$\left(\frac{x}{a}\right)^2 + 6\left(\frac{x}{a}\right) - 16 = 0$$

$$\left(\frac{x}{a} + 8\right)\left(\frac{x}{a} - 2\right) = 0$$

$$x = -8a \text{ or } x = 2a$$



$$\therefore x = 2a$$

$$y = \pm 2\sqrt{3}a \quad (y^2 = 12a^2 \quad y = \pm 2\sqrt{3}a)$$

$\therefore$ Required Area = Area OACB

$$= 2 \times \int_{-2\sqrt{3}a}^{2\sqrt{3}a} (x_1 - x_2) dy$$

$$= 2 \times \int_{-2\sqrt{3}a}^{2\sqrt{3}a} \sqrt{16a^2 - y^2} dy = 2 \times \int_{-2\sqrt{3}a}^{2\sqrt{3}a} \frac{y^2}{6a} dy$$

$$= \frac{2 \times}{2 \times \frac{1}{2}} \left[\frac{16a^2 - y^2}{2} + 16a^2 \sin^{-1} \frac{y}{4a} \right]_{-2\sqrt{3}a}^{2\sqrt{3}a} = \frac{y^2}{9a} \Big|_{-2\sqrt{3}a}^{2\sqrt{3}a}$$

$$= 2\sqrt{3}a \times 2a + 16a^2 \sin^{-1} \frac{\sqrt{3}}{2} - 0 = \frac{8 \times 2\sqrt{3}a^2}{\sqrt{3}}$$

$$= 4\sqrt{3}a^2 - \frac{8\sqrt{3}a^2}{3} + \frac{16a^2\pi}{3} \quad \left\{ \int \sqrt{A^2 - x^2} dx = \frac{x}{2} \sqrt{A^2 - x^2} + \frac{A^2}{2} \sin^{-1} \frac{x}{A} \right\}$$

$$= \frac{4}{\sqrt{3}}a^2 + \frac{16a^2\pi}{3} \quad \text{sq units}$$

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

$f(x)$ is increasing strictly in an interval if $f'(x) > 0$ in that interval
 $f(x)$ is strictly decreasing in an interval if $f'(x) < 0$ in that interval

$$f'(x) = 4x^3 - 24x^2 + 44x - 24$$

$$= 4(x^3 - 6x^2 + 11x - 8)$$

Maximum & Minimum values of $f(x) = \sec x + \log \cos x$

$$f'(x) = 0$$

$$\sec x \tan x + \frac{2}{\cos x}(-\sin x) = 0 \quad x \neq \frac{\pi}{2}$$

$$\therefore \tan x (\sec x - 2) = 0$$

$$\therefore x = \pi \quad \text{or} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Now $f''(x) < 0$ for maximum

$f''(x) > 0$ for minimum

$$f''(x) = \sec x (\sec x - 2) + \tan x \sec x$$

$$f''(\pi) = 1 \times -3 + 0 = -3 < 0$$

$$f''\left(\frac{\pi}{3}\right) = f''\left(\frac{5\pi}{3}\right) = 0 + 3 \times 2 = 6 > 0$$

Function attains maximum value at $x = \pi$ & minimum

$\therefore$ value at $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$$f(\pi) = -1$$

$$f\left(\frac{\pi}{3}\right) = 2 - 2 \log 2$$

But when $x = \pi$, function becomes undefined.

~~Maximum & Minimum~~ do not exist as Minimum -

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

Proof: $C_1 \rightarrow C_1 - 2C_2$

$$\Delta = \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ac \\ a^2+b^2 & c^2 & ab \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2$$

Now taking $a^2+b^2+c^2$ common from C_1 ,

$$\Delta = (a^2+b^2+c^2) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ac \\ 1 & c^2 & ab \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\Delta = (a^2 + b^2 + c^2) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2 - a^2 & ac - bc \\ 0 & c^2 - a^2 & ab - bc \end{vmatrix}$$

Taking $(a-b)$ common from R_2 & $(c-a)$ common from C_3

$$\Delta = (a^2 + b^2 + c^2)(a-b)(c-a) \begin{vmatrix} 1 & a^2 & bc \\ 0 & -(a+b) & c \\ 0 & a+c & -b \end{vmatrix}$$

Expanding along C_3

$$\begin{aligned} \Delta &= (a^2 + b^2 + c^2)(a-b)(c-a) \left(-(a+b) \cdot b + (a+c)c \right) \\ &= (a^2 + b^2 + c^2)(a-b)(c-a) (b^2 - c^2 + ac - ba) \\ &= (a^2 + b^2 + c^2)(a-b)(b-c)(c-a)(a+b+c) \end{aligned}$$

$$P.S. \quad (b-c)(a+b+c) = ab + b^2 + bc - ac - bc - c^2$$

$$b^2 - c^2 + ab - ac$$

25. Equation of plane containing two parallel lines

has DRS of perpendicular

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b}$$

$\vec{b}$ = DRS of the line

$\vec{a}_1$ = Point (Position vector on line 1)

$\vec{a}_2$ = Point (Position vector on line 2)

$$\vec{a}_1 = \hat{i} - \hat{j}$$

$$\vec{a}_2 = 2\hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 3\hat{j} - \hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 8\hat{i} + \hat{j} - 5\hat{k}$$

$$\therefore \text{Equation of plane} = 8x + y - 5z = 8 \times 1 + \hat{j} \times (-1) - 5 \times 0$$

$$8x + y - 5z = 7$$

$$\text{Now consider the line } \frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{1}$$

$$8 \times 3 + 1 \times 1 - 5 \times 5 = 0$$

$\therefore$ DRS of line & perpendicular to plane are perpendicular.

& Point on line = (2, 1, 2)

Equation of plane. $\therefore$ The line is contained in the plane.

$$f(x) = 4x^2 + 12x + 15$$

$$\text{Let } f(x) = y$$

$$y = 4x^2 + 12x + 15$$

$$4x^2 + 12x + 15 - y = 0$$

$$x = \frac{-12 \pm \sqrt{144 - 16(15-y)}}{8} = \frac{-12 \pm \sqrt{9-15+y}}{2}$$

$$\text{Consider } g(x) = \frac{\sqrt{x-6-3}}{2}$$

$$\text{Now } f \circ g(x) = f(g(x)) = f\left(\frac{\sqrt{x-6-3}}{2}\right) = 4\left(\frac{\sqrt{x-6-3}}{2}\right)^2 + 6(\sqrt{x-6-3}) + 15$$

$$= x-6-3-6\sqrt{x-6-3}-18+15$$

$$= x$$

$$g \circ f(x) = g(f(x)) = g(4x^2 + 12x + 15) = \frac{\sqrt{4x^2 + 12x + 9} - 3}{2} = \frac{2x+3-3}{2} = x$$

$$\text{Thus } f \circ g(x) = g \circ f(x) = x$$

By definition, q is the inverse of f .
 As inverse exists, f is invertible &
 $\Rightarrow f'(x) = \frac{\sqrt{x-6}-3}{2}$

$$f'(31) = \frac{\sqrt{31-6}-3}{2} = \frac{5-3}{2} = 1$$

$$f'(37) = \frac{\sqrt{37-6}-3}{2} = \frac{9-3}{2} = 3$$

$X:$	1	2	3	4	5	6
$P(X=)$	$\frac{10-1}{20} = \frac{1}{20}$	$\frac{6-3}{20} = \frac{3}{20}$	$\frac{3}{20}$	$\frac{1}{20}$	0	0

(as 5 & 6 can't
be minimum
values)

$$P(X=1) = \frac{5C_2}{6C_3}$$

$$P(X=2) = \frac{4C_2}{6C_3}$$

$$P(X=3) = \frac{3C_2}{6C_3}$$

$$P(X=2) = \frac{2C_2}{6C_3}$$

$$= \frac{10}{20}$$

$$= \frac{6}{20}$$

$$= \frac{3}{20}$$

$$= \frac{1}{20}$$

(Numbers from 2-6
can be chosen)
(Any two)

(Numbers from 3-6
can be chosen)
(Any two)

(Any 2 out of
1, 3, 6
can be chosen)
only 3/6
can be
chosen

$$\text{Mean} = E(x) = \sum x_i p_i$$

$$= \frac{10 \times 1}{20} + \frac{6 \times 2}{20} + \frac{3 \times 3}{20} + \frac{1 \times 4}{20} + 0 + 0$$

$$= \frac{10}{20} + \frac{12}{20} + \frac{9}{20} + \frac{4}{20} = \frac{35}{20} = 1.75$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

$$= \frac{10 \times 1}{20} + \frac{6 \times 4}{20} + \frac{3 \times 9}{20} + \frac{1 \times 16}{20} - \frac{49}{16}$$

$$= \frac{10 + 24 + 27 + 16}{20} - \frac{49}{16} = \frac{77}{20} - \frac{49}{16}$$

$$= \frac{7}{4} \left(\frac{-11}{5} - \frac{7}{4} \right)$$

$$= \frac{7}{4} \times \frac{9}{20}$$

$$\sigma^2 = \frac{63}{80}$$

80 //

$$\frac{44}{-33} \div 9$$

$$\frac{39}{20} \div \frac{16}{7}$$