

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
परीक्षार्थी प्रवेश-पत्र के अनुसार भरें

विषय Subject : MATHEMATICS					
विषय कोड Subject Code : 041					
परीक्षा का दिन एवं तिथि Day & Date of the Examination : MONDAY, 20.03.2017					
उत्तर देने का माध्यम Medium of answering the paper : ENGLISH					
प्रश्न पत्र के ऊपर लिखे कूड को दर्शाए : Write code No. as written on the top of the question paper :	<table border="1"> <tr> <th>Code Number</th> <th>Set Number</th> </tr> <tr> <td>65/1</td> <td>● (2) (3) (4)</td> </tr> </table>	Code Number	Set Number	65/1	● (2) (3) (4)
Code Number	Set Number				
65/1	● (2) (3) (4)				
अतिरिक्त उत्तर-पुस्तिका (ओं) की संख्या No. of supplementary answer-book(s) used					
विकलांग व्यक्ति : ☐ हाँ / नहीं Person with Disabilities : ☐ Yes / No No					
किसी शारीरिक अक्षमता से प्रभावित हो तो संबंधित वर्ग में ☒ का निशान लगाएँ। If physically challenged, tick the category B D H S C A					
B = दृष्टिहीन, D = मूक व बधिर, H = शारीरिक रूप से विकलांग, S = स्पास्टिक C = डिस्लेक्सिक, A = ऑटिस्टिक B = Visually Impaired, D = Hearing Impaired, H = Physically Challenged S = Spastic, C = Dyslexic, A = Autistic					
क्या लेखन - लिपिक उपलब्ध करवाया गया : हाँ / नहीं Whether writer provided : ☐ Yes / No No					
यदि दृष्टिहीन हैं तो उपयोग में लाए गये Visually challenged, name of software used : -					

*कृपया खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।
Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

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SECTION -4

$$A(\text{adj } A) = |A| I_n$$

$$A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$|A| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|A| = 8$$

$$2) \quad f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3} & ; x \neq 3 \\ k & ; x = 3 \end{cases}$$

$f(x)$ is continuous at $x = 3$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$$

$$\lim_{x \rightarrow 3} \frac{(x+3)^2 - 36}{x-3} = k$$

$$\lim_{x \rightarrow 3} \frac{(x+3-6)(x+3+6)}{(x-3)} = k$$

$$\lim_{x \rightarrow 3} \frac{(x-3)(x+9)}{(x-3)} = k$$

$$\lim_{x \rightarrow 3} (x+9) = k$$

$$12 = k$$

$$k = 12$$

$$3) \int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$

$$= -2 \int \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} dx$$

$$= (-2) \int \frac{\cos 2x}{\sin 2x} dx$$

$$= (-2) \int \cot 2x dx$$

$$= (-2) \log |\sin 2x| \times \frac{1}{2} + C$$

$$\int \cot x dx = \log |\sin x|$$

$$= -\log |\sin 2x| + C$$

$$= \log |\operatorname{cosec} 2x| + C$$

$$P_1 \Rightarrow 2x - y + 2z = 5$$

$$P_2 \Rightarrow 5x - 2.5y + 5z = 20$$

$$\Rightarrow 2x - y + 2z = \frac{20}{2.5} = \frac{20}{5} \times 2 = 8$$

$$\text{distance} = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right| = \left| \frac{8 - 5}{\sqrt{2^2 + (-1)^2 + 2^2}} \right| = \left| \frac{3}{\sqrt{4+1+4}} \right| = \left| \frac{3}{\sqrt{9}} \right| = \frac{3}{3} = 1 \text{ unit}$$

SECTION B:

5) A is skew symmetric

$$A = -A^T$$

$$|A| = (-1)|A^T|$$

$$|A| = -|A| \quad [\because |A| = |A^T|]$$

$$2|A| = 0$$

$$|A| = 0$$

$$\det A = 0$$

6) $f(x) = x^3 - 3x$
 $f'(x) = 3x^2 - 3$

$\because f(x)$ is a polynomial, it is continuous in the interval $[-\sqrt{3}, 0]$
 $\because f'(x)$ is a polynomial it is differentiable in the interval $(-\sqrt{3}, 0)$

$$f(0) = 0 - 3(0) = 0$$

$$f(-\sqrt{3}) = (-\sqrt{3})^3 - 3(-\sqrt{3}) = -3\sqrt{3} + 3\sqrt{3} = 0$$

$$f(0) = f(-\sqrt{3})$$

Hence there exists $c \in (-\sqrt{3}, 0)$ such that $f'(c) = 0$.

$$f'(c) = 3c^2 - 3 = 0$$

$$3c^2 - 3 = 0$$

$$3c^2 \Rightarrow 3(c^2 - 1) = 0$$

$$c^2 - 1 = 0$$

$$c = \pm 1$$

$+1$ doesn't exist between $(-\sqrt{3}, 0)$. Hence $c = -1$

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$3A = 3(x^2) \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{x^2}$$

$$S = 6x^2$$

$$\frac{dS}{dt} = 12x \frac{dx}{dt} = 12x \left(\frac{3}{x^2} \right) = \frac{36}{x}$$

$$\frac{dS}{dt} \Big|_{x=10\text{cm}} = \frac{36}{10} \text{ cm}^2/\text{sec} = 3.6 \text{ cm}^2/\text{sec}.$$

V : Volume of the cube of side x

S : Surface area of the cube of side x .

8) $f(x) = x^3 - 3x^2 + 6x - 100$

$$f'(x) = 3x^2 - 6x + 6$$

$$= 3(x^2 - 2x + 2)$$

discriminant of the formed quadratic = $b^2 - 4ac = (-2)^2 - 4(1)(2)$
 $= 4 - 8 = -4$

$$b^2 - 4ac < 0$$

but $a > 0$

$$[a = 1]$$

Hence $x^2 - 2x + 2 > 0$ for all $x \in \mathbb{R}$

$$\therefore 3(x^2 - 2x + 2) > 0 \quad ; x \in \mathbb{R}$$

$$f'(x) > 0 \quad ; x \in \mathbb{R}$$

Hence $f(x)$ is increasing on $\mathbb{R}$.

9) $P(2, 2, 1)$ $Q(5, 1, -2)$

Direction ratios of the line PQ = $(5-2), (1-2), (-2-1)$
 $= 3, -1, -3$

Equation of PQ

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$$

Any point on PQ is given by $(3\lambda+2, -\lambda+2, -3\lambda+1)$

$$3\lambda+2=4$$

$$3\lambda=2$$

$$\lambda = \frac{2}{3}$$

$$z \text{ coordinate} = -3\lambda+1 = -3\left(\frac{2}{3}\right)+1 = -2+1 = -1$$

$$y \text{ coordinate} = -\lambda+2 = -\frac{2}{3}+1 = \frac{1}{3}$$

Hence point is $(4, \frac{1}{3}, -1)$

$$\underline{z \text{ coordinate} = -1}$$

10) A: number obtained is even
= { 2, 4, 6 }

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

B: number obtained is red
= { 1, 2, 3 }

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

$A \cap B$ = number obtained is red and even
= { 2 }

$$P(A \cap B) = \frac{1}{6}$$

$$P(A)P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$P(A \cap B) \neq P(A)P(B)$ Hence events A and B are not independent events

11)

	Shirts	Trousers	Payment per day
(x) A	6	4	₹ 300
(y) B	10	4	₹ 400
	<u>60</u>	<u>32</u>	

Let A work for x and B for y days.

To Minimize : $Z = 300x + 400y$

Constraints : $6x + 10y \geq 60$

$4x + 4y \geq 32$

$x \geq 0$

$y \geq 0$

PQR is the solution.

P(5,3) $Z = 300(5) + 400(3) = ₹ 2700$

Q(0,8) $Z = 400(8) = ₹ 3200$

R(10,0) $Z = 300(10) = ₹ 3000$

Hence Z is minimum when A works for 5 days and B for 3 days.

1500
1200
1500
1200
2700

12) $\int \frac{dx}{5-8x-x^2}$

$$= \int \frac{dx}{21-16-2 \cdot x \cdot 4-x^2}$$

$$= \int \frac{dx}{21-(x^2+2 \cdot x \cdot 4+(4)^2)}$$

$$= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$$

$$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21}+x+4}{\sqrt{21}-x-4} \right| + C$$

$$= \frac{1}{2\sqrt{21}} \log \left| \frac{x+\sqrt{21}+4}{\sqrt{21}-4-x} \right| + C$$

$$\left[\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C \right]$$

SECTION C!

3)

$$\tan^{-1} \frac{x-3}{x-4} + \tan^{-1} \frac{x+3}{x+4} = \frac{\pi}{4}$$

$$\tan^{-1} \left(\frac{\frac{x-3}{x-4} + \frac{x+3}{x+4}}{1 - \left(\frac{x-3}{x-4} \right) \left(\frac{x+3}{x+4} \right)} \right) = \tan^{-1}(1)$$

$$\left[\because \tan^{-1} \left(\frac{x-3}{x-4} \right) \leq \frac{\pi}{4} \right.$$

$$\left. \tan^{-1} \left(\frac{x+3}{x+4} \right) \leq \frac{\pi}{4} \right]$$

$$\tan^{-1} \left(\frac{\frac{(x-3)(x+4) + (x+3)(x-4)}{x^2-16}}{\frac{x^2-16 - (x^2-9)}{x^2-16}} \right) = \tan^{-1}(1)$$

$$\therefore \tan^{-1}(A) + \tan^{-1}(B) = \tan^{-1} \left(\frac{A+B}{1-AB} \right)$$

$$\tan^{-1} \left(\frac{x^2 - \cancel{3x} + 4x - 12 + x^2 + \cancel{3x} - 4x - 12}{x^2 - 16 - x^2 + 9} \right) = \tan^{-1}(1)$$

$$\tan^{-1} \left(\frac{2x^2 - 24}{-7} \right) = \tan^{-1}(1)$$

$$\frac{2x^2 - 24}{-7} = 1$$

$$\cancel{2x^2} - 7 \quad 2x^2 - 24 = -7$$

$$2x^2 = -7 + 24 = 17$$

$$x^2 = \frac{17}{2} \Rightarrow x = \pm \sqrt{\frac{17}{2}}$$

$$\sqrt{\frac{17}{2}} < \sqrt{\frac{18}{2}}$$

$$\sqrt{\frac{17}{2}} < 3$$

$$\text{If } x = +\sqrt{\frac{17}{2}}$$

$$\frac{x-3}{x-4} = \text{positive}$$

$$\frac{x+3}{x+4} = \text{positive}$$

$$\text{If } x = -\sqrt{\frac{17}{2}}$$

$$\frac{x-3}{x-4} = \text{positive}$$

$$\frac{x+3}{x+4} = \text{positive}$$

Hence for $x = \pm\sqrt{\frac{17}{2}}$ $\tan^{-1} \frac{x-3}{x-4}$ and $\tan^{-1} \left(\frac{x+3}{x+4} \right)$ lie in first quadrant.

$$14) \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+2a-2a-1 & 2a+1 & 1 \\ 2a+1-a-2 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$

$$[C_1 \rightarrow C_1 - C_2]$$

$$= \begin{vmatrix} a^2-1 & 2a+1 & 1 \\ a-1 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= (a-1) \begin{vmatrix} a+1 & 2a+1 & 1 \\ 1 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$

$$[\text{Taking } (a-1) \text{ common out of } C_1]$$

$$= (a-1) \begin{vmatrix} a+1 & 2a-2 & 0 \\ 1 & a-1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$$

$$\begin{bmatrix} R_1 \rightarrow R_1 - R_3 \\ R_2 \rightarrow R_2 - R_3 \end{bmatrix}$$

✱

$$= (a-1) \begin{vmatrix} a+1 & 2(a-1) & 0 \\ 1 & (a-1) & 0 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= (a-1)^2 \begin{vmatrix} a+1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$$

[Taking (a-1) common out of C₂]

$$= (a-1)^2 \begin{vmatrix} a+1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= (a-1)^2 (a+1-2)$$

$$= (a-1)^2 (a-1)$$

$$= (a-1)^3$$

[On expanding the determinant]

Proved.

5)

$$e^y (x+1) = 1.$$

$$e^y = \frac{1}{x+1}.$$

①

On differentiating both sides w.r.t x .

$$e^y \frac{dy}{dx} = \frac{d}{dx} (x+1)^{-1}.$$

$$e^y \frac{dy}{dx} = -1 (x+1)^{-1-1}$$

$$e^y \frac{dy}{dx} = \frac{-1}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{-1}{(x+1)^2 e^y} = \frac{-1}{(x+1)^2} \frac{(x+1)}{1} = \frac{-1}{(x+1)}$$

②

On differentiating ② w.r.t x on both sides.

$$\frac{d^2y}{dx^2} = - \frac{d}{dx} (x+1)^{-1} = -1(x+1)^{-2} = \frac{1}{(x+1)^2}$$

③

$$\frac{d^2y}{dx^2} = \left(\frac{-1}{x+1} \right)^2 = \left(\frac{dy}{dx} \right)^2$$

[Using equation ②]

$$\text{Hence } \frac{d^2y}{dx^2} = \left(\frac{dy}{dx} \right)^2$$

Proved.

$$16) \quad I = \int \frac{\cos \theta}{(4 + \sin^2 \theta)(5 - 4 \cos^2 \theta)} d\theta.$$

$$= \int \frac{\cos \theta}{(4 + \sin^2 \theta)(5 - 4(1 - \sin^2 \theta))} d\theta.$$

$$= \int \frac{\cos \theta}{(4 + \sin^2 \theta)(1 + 4 \sin^2 \theta)} d\theta$$

$$= \int \frac{dt}{(4 + t^2)(1 + 4t^2)}$$

$$\begin{aligned} \text{let } \sin \theta &= t \\ \cos \theta d\theta &= dt \end{aligned}$$

$$\frac{1}{(4 + t^2)(1 + 4t^2)} = \left[\frac{(1 + 4t^2) - 4(4 + t^2)}{(4 + t^2)(1 + 4t^2)} \right] \times \frac{1}{-15}$$

$$= \frac{-1}{15} \left(\frac{1}{4 + t^2} - \frac{4}{1 + 4t^2} \right)$$

$$= \frac{4}{15} \left(\frac{1}{1 + 4t^2} \right) - \frac{1}{15} \left(\frac{1}{4 + t^2} \right)$$

$$I = \int \left[\frac{4}{15} \left(\frac{1}{4t^2+1} \right) - \frac{1}{15} \left(\frac{1}{4+t^2} \right) \right] dt$$

$$= \frac{4}{15} \int \frac{1}{1+4t^2} dt - \frac{1}{15} \int \frac{1}{2^2+t^2} dt$$

$$= \frac{1}{15} \int \frac{1}{\left(\frac{1}{2}\right)^2+t^2} dt - \frac{1}{15} \int \frac{1}{2^2+t^2} dt$$

$$= \frac{1}{15} \times \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{t}{\frac{1}{2}}\right) - \frac{1}{15} \times \frac{1}{2} \tan^{-1}\left(\frac{t}{2}\right) + C$$

$$= \frac{2}{15} \tan^{-1}(2t) - \frac{1}{30} \tan^{-1}\left(\frac{t}{2}\right) + C$$

$$= \frac{2}{15} \tan^{-1}(2\sin\theta) - \frac{1}{30} \tan^{-1}\left(\frac{\sin\theta}{2}\right) + C$$

$$= \frac{1}{30} \left(4 \tan^{-1}(2\sin\theta) - \tan^{-1}\left(\frac{\sin\theta}{2}\right) \right) + C$$

$$\left[\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \right]$$

17). $f(x) = |x-1| + |x-2| + |x-4|$

$$= \begin{cases} (x-1) - (x-2) - (x-4) & ; 1 \leq x < 2 \\ (x-1) + (x-2) - (x-4) & ; 2 \leq x \leq 4 \end{cases}$$

1.5

+ve -ve -ve

3

+ve +ve -ve

$$= \begin{cases} x-1-x+2-x+4 & ; 1 \leq x < 2 \\ x-1+x-2-x+4 & ; 2 \leq x \leq 4 \end{cases}$$

$$= \begin{cases} 5-x & ; 1 \leq x < 2 \\ x+1 & ; 2 \leq x \leq 4 \end{cases}$$

$$I = \int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$$

$$= \int_1^2 (5-x) dx + \int_2^4 (x+1) dx$$

$$= \left[5x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} + x \right]_2^4$$

$$\left[\because \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \text{ when } a \leq c \leq b \right]$$

$$= [10 - 2] - [5 - \frac{1}{2}] + [8 + 4] - [2 + 2]$$

$$= 8 - \frac{9}{2} + 12 - 4$$

$$= \frac{16 - 9}{2}$$

$$= \frac{23}{2} = 11.5$$

$$\begin{array}{r} 2 \times 12 \\ = 24 \\ 32 - 9 \\ = 23 \\ \hline 2 \overline{) 23} \\ 20 \\ \hline 3 \\ 2 \\ \hline 1 \\ 2 \\ \hline 3 \\ 2 \\ \hline 1 \end{array}$$

$$18) (\tan^{-1}x - y)dx = (1+x^2)dy$$

$$\left(\frac{\tan^{-1}x - y}{1+x^2} \right) = \frac{dy}{dx}$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1}x}{1+x^2}$$

$$\frac{dy}{dx} + Py = Q$$

Hence it's a linear differential equation solution
 $P = \frac{1}{1+x^2}$

$$\text{Integration factor} = e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1}x}$$

$$e^{\tan^{-1}x} \left(\frac{dy}{dx} + \frac{y}{1+x^2} \right) = e^{\tan^{-1}x} \frac{\tan^{-1}x}{1+x^2}$$

[On multiplying both sides by integration factor]

$$\frac{d}{dx} (y e^{\tan^{-1}x}) = e^{\tan^{-1}x} \frac{\tan^{-1}x}{1+x^2}$$

$$y e^{\tan^{-1}x} = \int e^{\tan^{-1}x} \frac{\tan^{-1}x}{1+x^2} dx + C$$

$$\text{Let } \int e^{\tan^{-1}x} \frac{\tan^{-1}x}{1+x^2} dx \text{ be } I_1$$

$$\tan^{-1}x = t$$

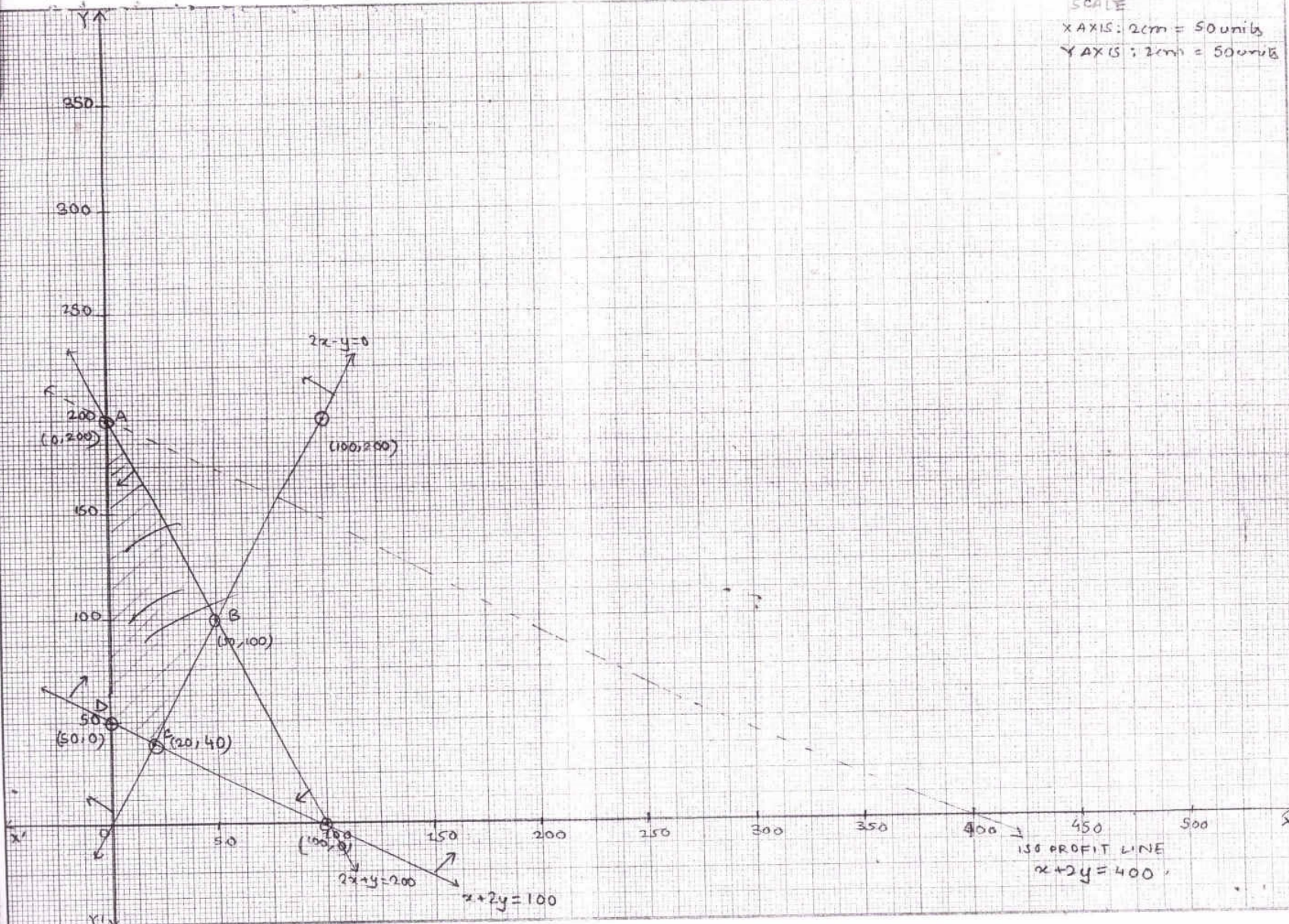
$$\frac{1}{1+x^2} dx = dt$$

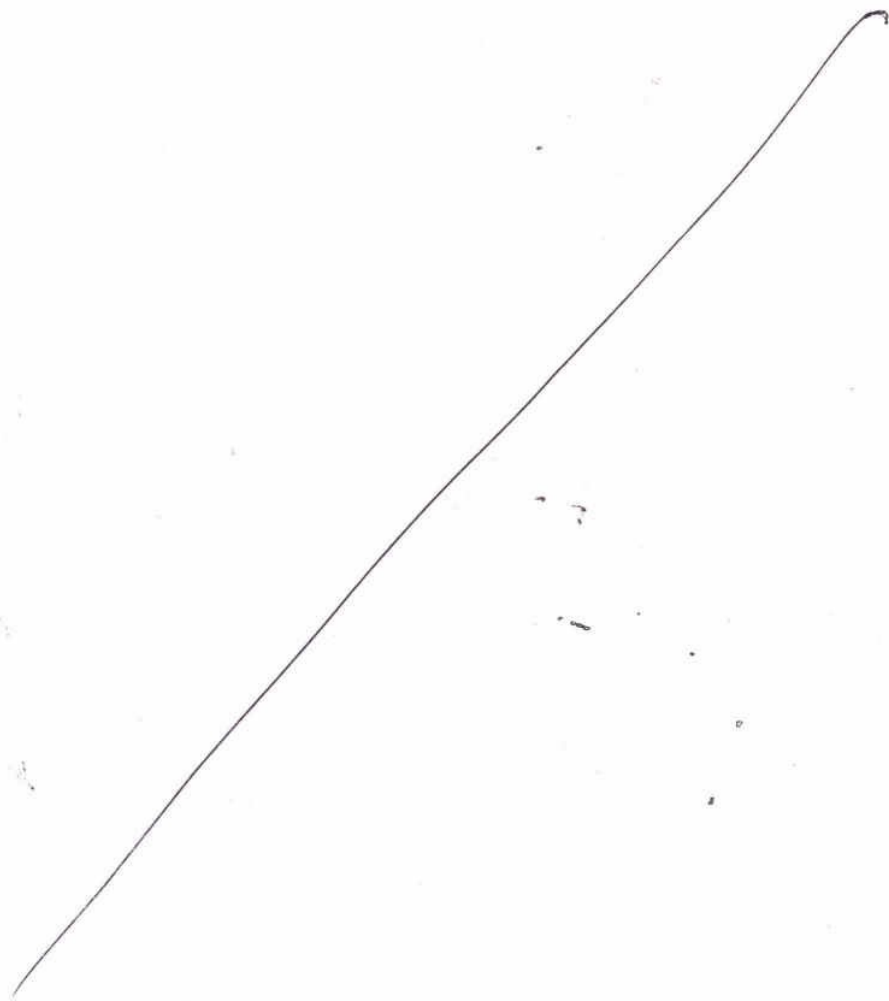
$$I_1 = \int e^t t dt$$

SCALE

X AXIS: 2cm = 50 units

Y AXIS: 2cm = 50 units

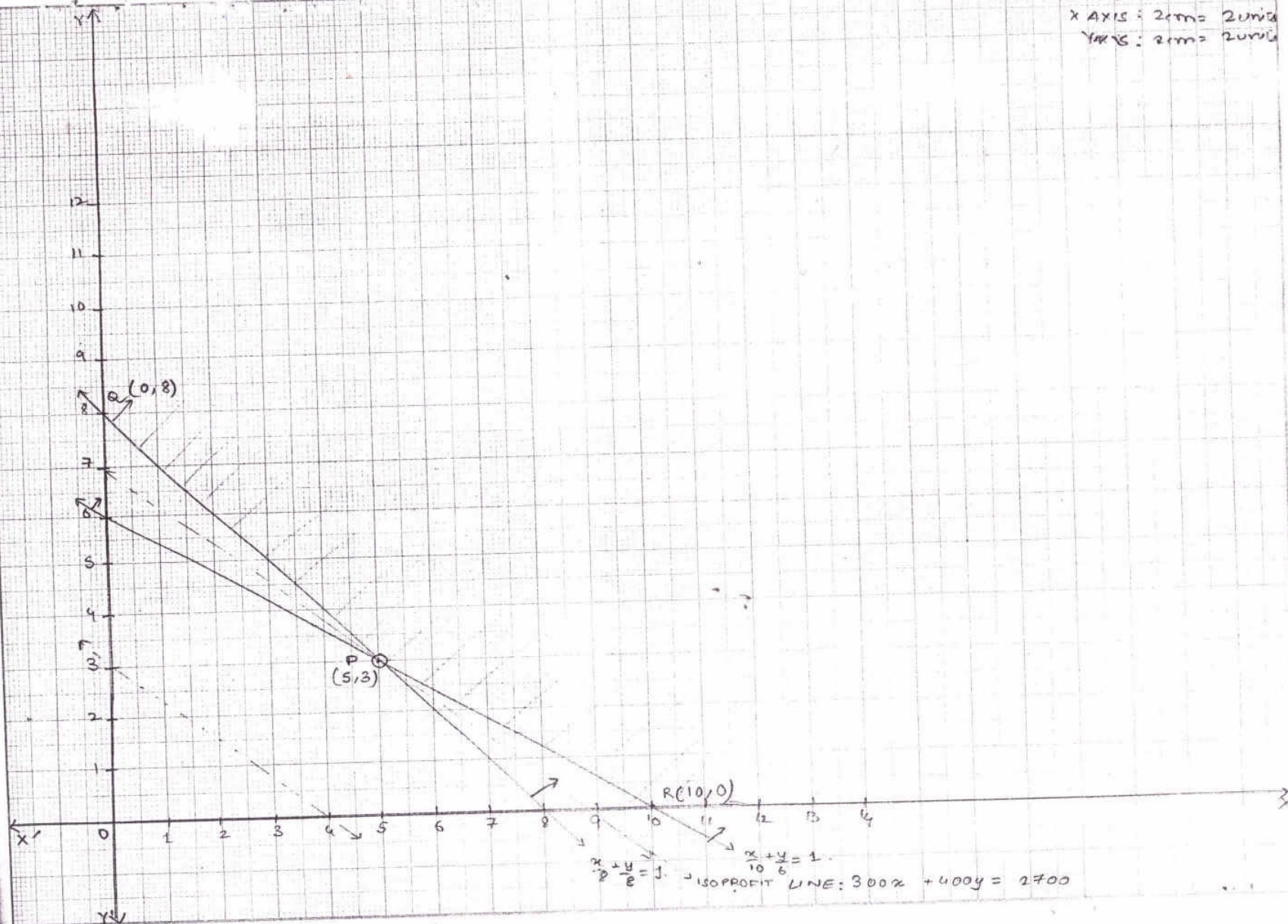




SCALE:

X AXIS: 2cm = 2 units

Y AXIS: 2cm = 2 units



$$I_1 = \int_{\frac{\pi}{2}}^t e^t t dt$$

$$= t \int e^t dt - \int \left(\frac{d(t)}{dt} \int e^t dt \right) dt$$

$$= t e^t - \int 1 \cdot e^t dt$$

$$= t e^t - e^t$$

$$= e^t (t - 1)$$

$$= e^{\tan^{-1} x} (\tan^{-1} x - 1)$$

$$\therefore y e^{\tan^{-1} x} = e^{\tan^{-1} x} (\tan^{-1} x - 1) + C$$

$$y = (\tan^{-1} x - 1) + \frac{C}{e^{\tan^{-1} x}}$$

$$y = \tan^{-1} x - 1 + C e^{-\tan^{-1} x}$$

$$\begin{aligned}
 \vec{OA} &= 2\hat{i} - \hat{j} + \hat{k} \\
 \vec{OB} &= \hat{i} - 3\hat{j} - 5\hat{k} \\
 \vec{OC} &= 3\hat{i} - 4\hat{j} - 4\hat{k}
 \end{aligned}$$

$$\begin{aligned}
 \vec{AB} &= \vec{OB} - \vec{OA} \\
 &= -\hat{i} - 2\hat{j} - 6\hat{k}
 \end{aligned}$$

$$|\vec{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41} \text{ units}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = \hat{i} - 3\hat{j} - 5\hat{k} \quad |\vec{AC}| = \sqrt{1^2 + (-3)^2 + (-5)^2} = \sqrt{35} \text{ units}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = 2\hat{i} - \hat{j} + \hat{k} \quad |\vec{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6} \text{ units}$$

$$\vec{AC} \cdot \vec{BC} = 2 + 3 - 5 = 0 \quad \text{Hence } \vec{AC} \perp \vec{BC} \quad \text{Hence } \angle C = 90^\circ$$

$$|\vec{BC}|^2 + |\vec{AC}|^2 = |\vec{AB}|^2$$

Hence ~~the~~ ΔABC is right angled at C.

$$\text{area} = \frac{1}{2} |\vec{AC} \times \vec{BC}|$$

$$= \frac{1}{2} \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -5 \\ 2 & -1 & 1 \end{vmatrix} \right| = \frac{1}{2} | \hat{i}(-8) - \hat{j}(11) + \hat{k}(5) |$$

$$= \frac{1}{2} \sqrt{(-8)^2 + (-1)^2 + (5)^2} = \frac{1}{2} \sqrt{209} \text{ so. units}$$

$$= \frac{1}{2} \sqrt{210} \text{ so. units}$$

$$= \sqrt{\frac{210}{4}} = \sqrt{52.5} \text{ so. units}$$

$\begin{array}{r} 121 \\ 25 \\ \hline 146 \\ 64 \\ \hline 10 \end{array}$	$\begin{array}{r} 64 \\ 121 \\ 25 \\ \hline 209 \end{array}$
$\begin{array}{r} 52.5 \\ \times 4 \\ \hline 210.0 \end{array}$	$\begin{array}{r} 35 \\ \times 6 \\ \hline 210 \end{array}$

$$20) \vec{OA} = 3\hat{i} + 6\hat{j} + 9\hat{k}$$

$$\vec{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{OC} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{OD} = 4\hat{i} + 6\hat{j} + \lambda\hat{k}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = -2\hat{i} - 4\hat{j} - 6\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = -\hat{i} - 3\hat{j} - 8\hat{k}$$

$$\vec{AD} = \vec{OD} - \vec{OA} = \hat{i} + 0\hat{j} + (\lambda - 9)\hat{k}$$

$$\begin{aligned} &= \vec{a} \\ &= \vec{b} \\ &= \vec{c} \end{aligned}$$

Scalar triple product $[\vec{a} \vec{b} \vec{c}] = 0 \quad \therefore \vec{a}, \vec{b}, \vec{c} \text{ are coplanar.}$

$$\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & \lambda-9 \end{vmatrix} = 0$$

$$(-2)(-3\lambda+27) + 4(-\lambda+9+8) - 6(3) = 0.$$

$$6\lambda - 54 - 4\lambda + 68 - 18 = 0.$$

$$2\lambda = 54 + 18 - 68$$

$$2\lambda = 72 - 68 = 4$$

$$\lambda = 2$$

$$\begin{array}{r} 217 \\ \times 4 \\ \hline 868 \\ 80 \end{array}$$

21) x	P(x)	x P(x)	P(x) x^2
4	$P(1,3), (3,1)$ $= \frac{1 \times 1}{4 \times 3} + \frac{1 \times 1}{4 \times 3} = \frac{2}{12}$	$\frac{2 \times 4^2}{12 \times 3} = \frac{2}{3}$	$\frac{2}{3} \times 4 = \frac{8}{3}$
6	$P(1,5), (5,1)$ $= \frac{1 \times 1}{4 \times 3} + \frac{1 \times 1}{4 \times 3} = \frac{2}{12}$	$\frac{2 \times 6}{12} = 1$	$1 \times 6 = 6$
8	$P(1,7), (7,1), (3,5), (5,3)$ $= \frac{1 \times 1}{4 \times 3} + \frac{1 \times 1}{4 \times 3} + \frac{1 \times 1}{4 \times 3} + \frac{1 \times 1}{4 \times 3} = \frac{4}{12}$	$\frac{4 \times 8}{12 \times 3} = \frac{8}{3}$	$\frac{8 \times 8}{3} = \frac{64}{3}$
10	$P(3,7), (7,3)$ $= \frac{1 \times 1}{4 \times 3} + \frac{1 \times 1}{4 \times 3} = \frac{2}{12}$	$\frac{10 \times 2}{12 \times 6} = \frac{5}{3}$	$\frac{5 \times 10}{3} = \frac{50}{3}$
12	$P(5,7), (7,5)$ $= \frac{1 \times 1}{4 \times 3} + \frac{1 \times 1}{4 \times 3} = \frac{2}{12}$	$\frac{12 \times 2}{12} = 2$	$2 \times 12 = 24$

30

$72 \times 1 = 72$
 50
 $1 \times 20 = 20$
 3
 $= 4$

40.667
 70
 70.667
 64.000
 6.667

$$\bar{X} = \sum_{i=4}^{12} P(x_i) x_i$$

$$= \frac{2}{3} + 1 + \frac{8}{3} + 5 + 2$$

$$= 3 + \frac{15}{3} = 8$$

Mean = 8

$$\text{Variance} = \left(\sum_{i=4}^{12} P(x_i) x_i \right)^2 \rightarrow \sum_{i=4}^{12} P(x_i) x_i^2 - \left(\sum_{i=4}^{12} P(x_i) x_i \right)^2$$

$$= \frac{122}{3} + 30 - 64$$

$$= 40.667 + 30 - 64$$

$$= 70.667 - 64$$

$$= 6.667$$

2) E_1 : Event that students have 100% attendance

$$P(E_1) = \frac{30}{100}$$

E_2 : Event that students are irregular

$$P(E_2) = \frac{70}{100}$$

A: Student has grade A

$$P(A|E_1) = \frac{70}{100}$$

$$P(A|E_2) = \frac{10}{100}$$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{10}{100}}$$

$$= \frac{2100}{2800} = \frac{3}{4}$$

$$\begin{array}{r} 2100 \\ 700 \\ \hline 2800 \\ 21 = 213 \\ 21+7 = 284 \end{array}$$

Yes regularity is required in school for discipline as well as scoring well in academics.

23) $Z = x + 2y$

Constraints: $x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x \geq 0$

$y \geq 0$

$L_1 \Rightarrow x + 2y = 100$

$L_2 \Rightarrow 2x - y = 0$

$\frac{x}{100} + \frac{y}{50} = 1$

$y = 2x$

PG: $(100, 0)$ ~~$(50, 0)$~~ , $(0, 50)$

PG: $(50, 100)$, $(100, 200)$

$L_3 \Rightarrow 2x + y = 200$

$\frac{x}{100} + \frac{y}{200} = 1$

PG: $(100, 0)$, $(0, 200)$

Solution is in the region ABCD.

A	$(0, 200)$	$Z = x + 2y = 400$
B	$(50, 100)$	$Z = x + 2y = 50 + 200 = 250$
C	$(20, 40)$	$Z = x + 2y = 20 + 80 = 100$
D	$(50, 0)$	$Z = x + 2y = 50$

Hence Z is maximum when $x = 0$ $y = 200$ [at $(0, 200)$]

SECTION D:-

$$4) \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-4)1 + 4(1) + 4(2) & -4(-1) + 4(-2) + 4(1) & -4(1) + 4(-2) + 4(3) \\ -7(1) + 1(1) + 3(2) & -7(-1) + 1(-2) + 3(2) & -7(1) + 1(-2) + 3(3) \\ 5(1) - 3(1) - 1(2) & 5(-1) - 3(-2) - 1(1) & 5(1) - 3(-2) - 1(3) \end{bmatrix}$$

$$\begin{array}{l} 7+2 \\ -6 \\ 7+2+3, \\ 1+6-5 \\ 5+6-3 \end{array}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

~~18/18~~

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} -4(4) + 4(9) + 4(1) \\ -7(4) + 1(9) + 3(1) \\ 5(4) - 3(9) - 1(1) \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} -16 + 36 + 4 \\ -28 + 9 + 3 \\ 20 - 27 - 1 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

Hence $x = 3$
 $y = -2$
 $z = -1$

$$A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$AA^{-1} = I$$

$$\frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$5) f: R - \left\{ -\frac{4}{3} \right\} \rightarrow R - \left\{ \frac{4}{3} \right\} \quad f(x) = \frac{4x+3}{3x+4}$$

$$\text{Let } f(x_1) = f(x_2)$$

$$\frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$$

$$(4x_1+3)(3x_2+4) = (4x_2+3)(3x_1+4)$$

$$12x_1x_2 + 9x_2 + 16x_1 + 12 = 12x_1x_2 + 9x_1 + 16x_2 + 12$$

$$9x_2 = 9x_1$$

$$x_1 = x_2$$

Hence $f(x)$ is one to one.

$$\text{Let } y = \frac{4x+3}{3x+4}$$

$$y(3x+4) = 4x+3$$

$$3xy + 4y - 4x = 3$$

$$x(3y-4) = 3-4y$$

$$x = \frac{3-4y}{3y-4}$$

$$(3y-4)$$

Hence for every $y \in R - \left\{ \frac{4}{3} \right\}$ there exists x such that
 $x \in R - \left\{ -\frac{4}{3} \right\}$. ~~then~~

$$\frac{-4}{3} = \frac{3-4y}{3y-4}$$

$$-12y + 16 = 9 - 12y$$

$$16 = 9$$

which is not possible ; Hence x cannot be $-\frac{4}{3}$.

Hence $f(x)$ is onto function.

$\therefore f(x)$ is a bijective. Hence f^{-1} exists

~~$f(f^{-1}(x)) = x$~~ By the definition of inverse

$$f(f^{-1}(x)) = x$$

$$4f^{-1}(x) + 3 = x.$$

$$3f^{-1}(x) + 4$$

$$4f^{-1}(x) + 3 = 3x f^{-1}(x) + 4x$$

$$f^{-1}(x)(4-3x) = 4x-3$$

$$f^{-1}(x) = \frac{4x-3}{4-3x} = \frac{3-4x}{3x-4}$$

$$f^{-1}(0) = \frac{3-4(0)}{3(0)-4} = \frac{3}{-4} = -\frac{3}{4}$$

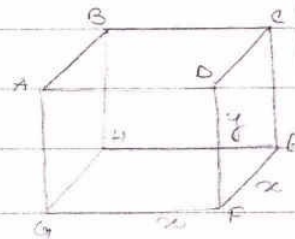
$$f^{-1}(x) = 2$$

$$\frac{3-4x}{3x-4} = 2 \Rightarrow 3-4x = 6x-8$$

$$10x = 11$$

$$x = \frac{11}{10}$$

Let a closed cuboid have a base of $x \times x$ and a height y
 Let its volume be V and surface area be S



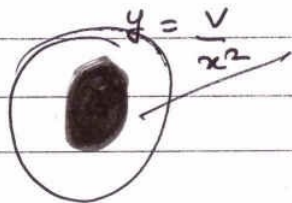
$$V = x^2 y$$

$$S = 2(x^2 + 2xy)$$

$$= 2x^2 + 4xy$$

$$= 2x^2 + 4x \frac{V}{x^2}$$

$$= 2x^2 + \frac{4V}{x}$$



$$\frac{dS}{dx} = \frac{d}{dx} \left(2x^2 + \frac{4V}{x} \right) = 4x + \frac{4V(-1)}{x^2} = 4x - \frac{4V}{x^2}$$

$$\frac{d^2S}{dx^2} = 4 - \frac{4V(-2)}{x^3} = 4 + \frac{8V}{x^3}$$

To maximize or minimize S , $\frac{dS}{dx} = 0$

$$4x - \frac{4V}{x^2} = 0$$

$$4x = \frac{4V}{x^2}$$

$$x^3 = V$$

$$x = V^{1/3}$$

$$\frac{d^2S}{dx^2} \bigg|_{x=V^{1/3}} = 4 + \frac{8V}{V} = 12 \text{ which is } > 0$$

Hence S is minimum at $x = V^{1/3}$

$$x = \sqrt[3]{V}$$

$$x^3 = V$$

$$x^3 = x^2 y$$

$$x^3 - x^2 y = 0$$

$$x^2(x - y) = 0$$

$$\therefore x = y$$

Hence the given cuboid is a cube of side x .

7)

$$4y = 3x^2$$

$$\Rightarrow y = \frac{3x^2}{4}$$



$$3x - 2y + 12 = 0$$

$$\Rightarrow 2y - 3x = 12$$

$$\Rightarrow \frac{y}{6} - \frac{x}{4} = 0$$

$$\Rightarrow \frac{y}{6} + \frac{x}{-4} = 0$$

$$\Rightarrow 2y = 3x + 12$$

To find points of intersection :-

$$4y = 3x^2$$

$$2(2y) = 3x^2$$

$$2(3x + 12) = 3x^2$$

$$6x + 24 = 3x^2$$

$$3x^2 - 6x - 24 = 0$$

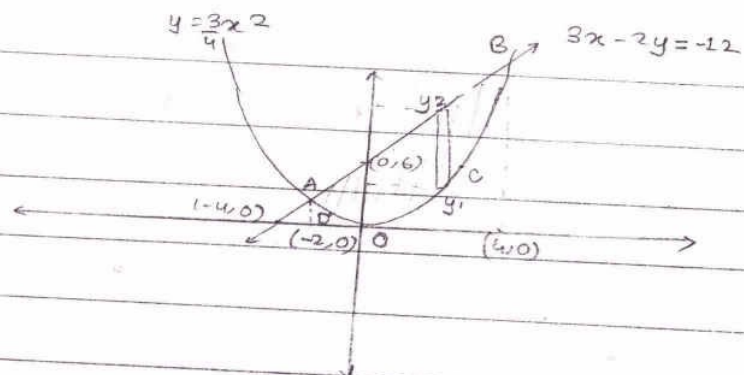
$$x^2 - 2x - 8 = 0$$

$$x^2 - 4x + 2x - 8 = 0$$

$$x(x-4) + 2(x-4) = 0$$

$$(x+2)(x-4) = 0$$

$$x = -2 \text{ or } 4$$



Hence of ODABCO

$$\text{area} = \int_{-2}^4 (y_2 - y_1) dx$$

$$= \int_{-2}^4 \frac{3x+12}{2} - \frac{3x^2}{4} dx$$

$$= \left[\frac{3}{2} \frac{x^2}{2} + 6x - \frac{3}{4} \frac{x^3}{3} \right]_{-2}^4$$

$$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4} \right]_{-2}^4$$

$$= \left[\frac{3(16)}{4} + 6(4) - \frac{16}{4} \right] - \left[\frac{3 \times 4}{4} - 12 - \frac{(-8)^2}{4} \right]$$

$$= [12 + 24 - 4] - [3 - 12 + 2]$$

$$= 20 + 7$$

$$= 27 \text{ sq. units}$$

$$= 27 \text{ sq. units}$$

$$b) (x-y) \frac{dy}{dx} = x+2y$$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\frac{dy}{dx} = \frac{1+2\left(\frac{y}{x}\right)}{1-\left(\frac{y}{x}\right)}$$

$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ Hence the solution of differential equation is that of a homogenous solution.

$$\text{Let } y = vx$$

$$v + x \frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$x \frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+2v-v-v^2}{1-v} = \frac{1+v+v^2}{1-v}$$

$$\frac{1-v}{1+v+v^2} dv = \frac{dx}{x}$$

On integrating both sides :-

$$\int \frac{1-v}{1+v+v^2} dv = \int \frac{dx}{x} + c$$

$$\int \frac{1-v}{1+v+v^2} dv = \ln|x| + c$$

$$\int \frac{1-v}{1+v+v^2} dv = \ln|x| + c$$

$$\frac{1}{2} \int \frac{2-2v}{1+v+v^2} dv = \ln|x| + c$$

$$-\frac{1}{2} \left[\int \frac{2v+1}{1+v+v^2} dv - 3 \int \frac{dv}{1+v+v^2} \right] = \ln|x| + c$$

$$-\frac{1}{2} \left[\int \frac{dv}{1+v+v^2} - 3 \int \frac{dv}{v^2 + 2 \cdot v \cdot \frac{1}{2} + \frac{1}{4} + \frac{3}{4}} \right] = \ln|x| + c$$

$$-\frac{1}{2} \left[\int \frac{dt}{t} - 3 \int \frac{dv}{\left(\frac{v+\frac{1}{2}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \right] = \ln|x| + c$$

$$\text{Let } 1+v+v^2 = t$$

$$(1+2v)dv = dt$$

$$-\frac{1}{2} \left[\ln|t| - 3 \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \right] = \ln|x| + c$$

$$\left[\because \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$-\frac{1}{2} \left[\ln|1+v+v^2| - 2\sqrt{3} \tan^{-1} \left(\frac{2v+1}{\sqrt{3}} \right) \right] = \ln|x| + c$$

$$-\frac{1}{2} \ln \left| 1 + \frac{y}{x} + \frac{y^2}{x^2} \right| + \sqrt{3} \tan^{-1} \left(\frac{2y+x}{\sqrt{3}x} \right) - \ln|x| = c$$

$$2\sqrt{3} \tan^{-1} \left(\frac{2y+x}{\sqrt{3}x} \right) = \ln|x^2| + \ln \left| 1 + \frac{y}{x} + \frac{y^2}{x^2} \right| + 2c$$

$$2\sqrt{3} \tan^{-1} \left(\frac{2y+x}{\sqrt{3}x} \right) = \ln |x^2 + xy + y^2| + c'$$

$$[c' = 2c]$$

$$y=0, x=1$$

$$2\sqrt{3} \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \ln |1| + c'$$

$$30^\circ = \frac{20}{6}$$

$$c' = 2\sqrt{3} \times \frac{\pi}{6} = \frac{\sqrt{3}\pi}{3} = \frac{\pi}{\sqrt{3}}$$

Hence ;

$$2\sqrt{3} \tan^{-1} \left(\frac{2y+x}{\sqrt{3}x} \right) = \ln |x^2 + xy + y^2| + \frac{\pi}{\sqrt{3}}$$

29)

~~Let the plane~~

$$A (a, 0, 0)$$

$$B (0, b, 0)$$

$$C (0, 0, c)$$

Then centroid of $\triangle ABC$ is $G \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3} \right)$.

~~Let the~~ The equation of the plane is given by :-

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$$(x-a)(bc) - y(-ac) + z(ab) = 0$$

$$xbc - abc + acy + abz = 0$$

$$bcx + acy + abz - abc = 0 \Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0$$

distance of the origin from the plane = $3p$

$$\left| \frac{-1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \right| = 3p$$

$$\frac{1}{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} = 9p^2$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2}$$

Let the centroid G be at (x, y, z)
 $x = \frac{a}{3}$ $y = \frac{b}{3}$ $z = \frac{c}{3}$

$$\left(\frac{3}{a}\right)^2 + \left(\frac{3}{b}\right)^2 + \left(\frac{3}{c}\right)^2 = \frac{1}{p^2}$$

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$$

Proved. $\therefore$

Hence locus of G is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$

Paul
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