Solved Question Paper 2017
Class 10
Summative Assessment II
Subject - Mathematics

Time allowed: 3 hours **Maximum Marks: 90**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 31 questions divided into four sections - A, B, C and D.

(iii) Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.

(iv) Use of calculators is not permitted.

SECTION A

Question 1. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?

Solution.

Let first term of AP be a and d be its common difference.
Now, nth term of an AP is given as:

$a_n = a + (n-1)d$

$\Rightarrow \quad a_{21} = a + (21-1)d = a + 20d$

and $\quad a_7 = a + (7-1)d = a + 6d$
According to the question:

$a_{21} - a_7 = 84$

$\Rightarrow \quad (a+20d) - (a+6d) = 84$

$\Rightarrow \quad a + 20d - a - 6d = 84$

$\Rightarrow \quad 14d = 84$

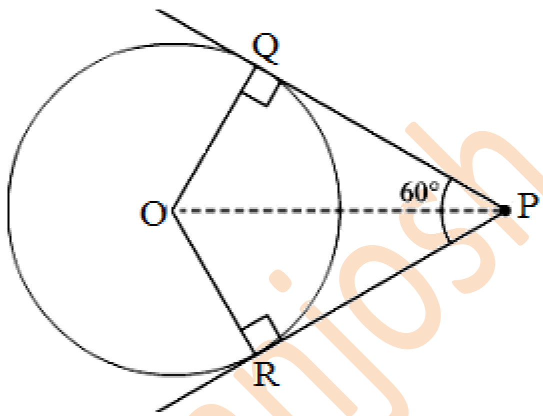
$\Rightarrow \quad d = 6$

Thus, the required common difference of the AP is 6.

Question 2. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

Solution.

Let PQ and PR be the two tangents drawn to the circle with radius a and centre O such that ∠QPR = 60°.



In ΔOPQ and ΔOPR

OQ = OR = Radius

∠OQP = ∠ORP = 90° (Tangents drawn to a circle are perpendicular to radius at the point of contact)

PQ = PR (Lengths of tangents drawn from an external point to the circle are equal)

So, ΔOPQ ≅ ΔOPR (SAS Congruence Axiom)

∴ ∠ OPQ = ∠ OPR = 30° (CPCT)

Now, in ΔOPQ,

$$\sin 30^\circ = \frac{OQ}{OP}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{a}{OP} \quad [OQ = \text{Radius} = a\,(\text{given})]$$

$$\Rightarrow \quad OP = 2a$$

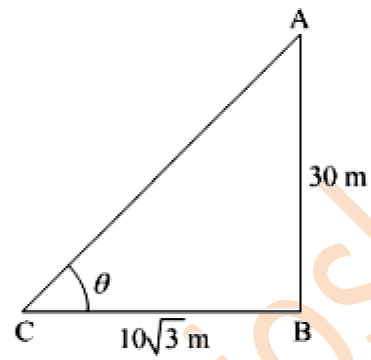
Thus, the length of OP is $2a$.

Question 3. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun?

Solution.

Let AB be the 30 m high tower and BC = $10\sqrt{3}$ m be the length of its shadow on ground.

Let the angle of elevation of the sun from the ground be θ.



Now, in ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{30}{10\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{30\sqrt{3}}{10\times 3}$$

$$\Rightarrow \quad \tan\theta = \frac{30\sqrt{3}}{30} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60^{\circ}$$

$$\Rightarrow \quad \theta = 60^{\circ}$$

Thus, the angle of elevation of the sun is 60°.

Question 4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap?

Solution.

We know that Probability of an event (E) = $\dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

Let E be the event of selecting rotten apple.
Let n be the number of rotten apples in the heap.

$$P(E) = \frac{n}{900}$$

$$\Rightarrow \quad 0.18 = \frac{n}{900}$$

$$\Rightarrow \quad n = 162$$

Thus, total rotten apples = 162.

SECTION B

Question 5. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

Solution.

Given equation can be written as:

$$x^2 - \frac{14}{p}x + \frac{8}{p} = 0 \qquad \text{....(i)}$$

Let the first root be α, then the second root will be 6α.

General equation is given as:

$$ax^2 + bx + c = 0 \qquad \text{....(ii)}$$

Now, sum of roots = $\dfrac{-b}{a}$

$$\Rightarrow \quad \alpha + 6\alpha = \frac{14}{p} \qquad \text{[Comparing (i) and (ii)]}$$

$$\Rightarrow \quad 7\alpha = \frac{14}{p}$$

$\Rightarrow \quad \alpha = \frac{2}{p}$

Also, product of roots = *ca*

$\Rightarrow \quad \alpha \times 6\alpha = \frac{8}{p}$ [From (i) and (ii)]

$\Rightarrow \quad 6\alpha^2 = \frac{8}{p}$

$\Rightarrow \quad 6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$

$\Rightarrow \quad p = 3$

Hence, the value of p is 3.

Question 6. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$ is the first negative term?

Solution.

The given sequence is $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$

Here, First term, $a = 20$

And common difference, $d = 19\frac{1}{4} - 20 = \frac{77-80}{4} = -\frac{3}{4}$

Let the *n*th term of the AP be the first negative term.

$\Rightarrow a_n < 0$

$\Rightarrow a + (n-1)d < 0$

$\Rightarrow 20 + (n-1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow \frac{83}{4} - \frac{3n}{4} < 0$

$\Rightarrow 83 - 3n < 0$

$\Rightarrow 3n > 83$

$\Rightarrow n > 27\frac{2}{3}$

$\Rightarrow n \geq 28$

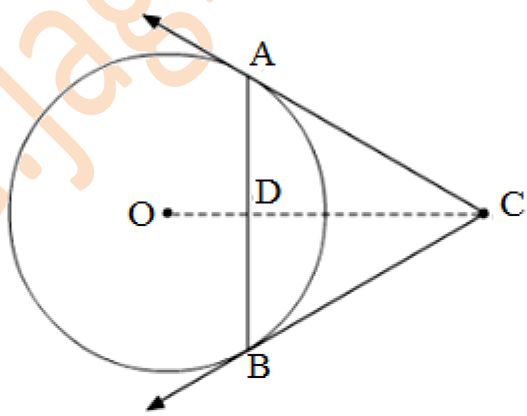
Thus, the 28th term is the first negative term of the given AP.

Question 7. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Solution.

Let AB be a chord of a circle with centre O, and let AC and BC to the same circle.

Join OC to cut AB at D.



We have to prove that ∠CAD = ∠CBD.

As the line segment joining the centre to the external point from where tangents are drawn, bisects the angle between two tangents.

So, ∠ACD = ∠BCD ....(i)

In ΔACD and ΔBCD, we have

CA = CB [Tangents from an external point are equal]

$\angle ACD = \angle BCD$ [From (i)]

CD = CD [Common]

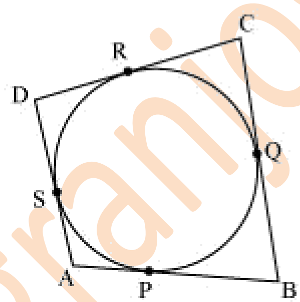
$\therefore\ \Delta ACD \cong \Delta BCD$ [By SAS congruency rule]

$\Rightarrow \angle CAD = \angle CBD$ [BY CPCT]

Question 8. A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA.

Solution.

ABCD is a quadrilateral. Suppose a circle touches the sides AB, BC, CD and DA of the quadrilateral ABCD at P, Q, R and S, respectively.



We know that the length of tangents drawn from an external point to a circle are equal.

DR = DS .....(i)

CR = CQ .....(ii)

BP = BQ .....(iii)

AP = AS .....(iv)

Adding (i), (ii), (iii) and (iv), we get

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

Or, AB + CD = BC + DA

Question 9. A line intersects the *y*-axis and *x*-axis at the points P and Q respectively. If (2, –5) is the mid-point of PQ, then find the coordinates of P and Q.

Solution.

Suppose the line intersects the *y*-axis at P(0, *y*) and the *x*-axis at Q(*x*, 0).

It is given that (2, –5) is the mid-point of PQ.
Using mid-point formula, we have:

$$\left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \left(\frac{x}{2}, \frac{y}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \frac{x}{2} = 2 \text{ and } \frac{y}{2} = -5$$

$$\Rightarrow \quad x = 4, y = -10$$

Thus, the coordinates of P and Q are (0, −10) and (4, 0), respectively.

Question 10. If the distances of P(*x*, *y*) from A(5, 1) and B(–1, 5) are equal, then prove that 3*x* = 2*y*.

Solution.

Given: PA = PB
To Prove: 3*x* = 2*y*
Proof:

Since PA = PB
Thus, according to the distance formula,

$$\sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$$

$$\Rightarrow \quad (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2 \text{ (Squaring both sides)}$$

$$\Rightarrow \quad x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 - 10y + 25$$

$$\Rightarrow \quad -10x - 2y = 2x - 10y$$

$$\Rightarrow \quad 8y = 12x$$

$$\Rightarrow \quad 3x = 2y$$

Hence, $3x = 2y$.

SECTION C

Question 11. If *ad* ≠ *bc*, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

Solution.

The given equation is $(a^2 + b^2)x^2 + 2\,(ac + bd)x + (c^2 + d^2) = 0$
We know, $D = b^2 - 4ac$
Thus,
$D = [2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2)]$

$= [4(a^2c^2+b^2d^2+ 2abcd)] - 4(a^2+b^2)(c^2+d^2)$

$= \; 4[(a^2c^2+b^2d^2+ 2abcd) - (a^2c^2+a^2d^2+b^2c^2+b^2d^2)]$

$= 4[a^2c^2+b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$
$= 4[2abcd - b^2c^2 - a^2d^2]$

$= -4[a^2d^2+b^2c^2-2abcd]$

$= -4[ad-bc]^2$
But we know that $ad \neq bc$
Therefore, $(ad-bc) \neq 0$

$\Rightarrow \quad (ad-bc)^2 > 0$

$\Rightarrow \; -4(ad-bc)^2 < 0$

$\Rightarrow \quad D < 0$
Hence, the given equation has no real roots.

Question 12. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.

Solution.

Here, a_1 or $a = 5, a_n = 45$ and $S_n = 400$

We know that nth term of an AP is given as,

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 45 = 5 + (n-1)d$$

$$\Rightarrow \quad (n-1)d = 40 \quad ....(\text{i})$$

Now, sum of n terms is given as:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 400 = \frac{n}{2}[2 \times 5 + (n-1)d]$$

$$\Rightarrow \quad \frac{800}{n} = [10 + 40] \qquad [\text{Using (i)}]$$

$$\Rightarrow \quad n = \frac{800}{50} = 16$$

Putting $n = 16$ in equation (i), we get:

$$(16-1)d = 40$$

$$\Rightarrow \quad d = \frac{40}{15} = \frac{8}{3}$$

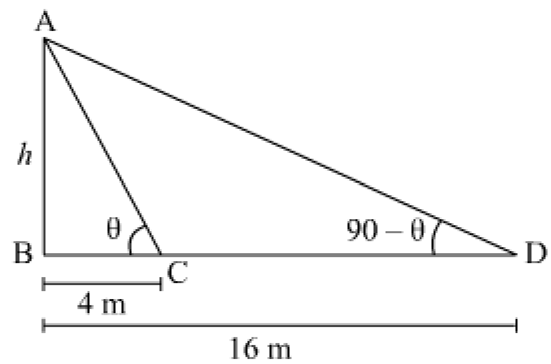
Hence, the common difference of the A.P. is $\frac{8}{3}$ and number of terms are16.

Question 13. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

Solution.

Let AB be the tower of height h m. Suppose the angle of elevation of the top of the tower from point C on the ground be θ. Then the angle of elevation of the top of the tower from point D is $90° - \theta$.

Here, BC = 4 m and BD = 16 m.



In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{4} \text{....(i)}$$

InΔABD,

$$\tan\left(90^{\circ} - \theta\right) = \frac{AB}{BD} = \frac{h}{16}$$

$$\Rightarrow \cot\theta = \frac{h}{16} \quad \text{.....(ii)}$$

Multiplying (i) and (ii), we get:

$$\tan\theta \times \cot\theta = \frac{h}{4} \times \frac{h}{16}$$

$$\Rightarrow 1 = \frac{h^2}{64}$$

$$\Rightarrow h^2 = 64$$

$$\Rightarrow h = \sqrt{64} = 8\text{m}$$

Thus, the height of the tower is 8 m.

Question 14. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

Solution.

Suppose number of black ball = x

It is given that White balls = 15

According to question,
P(Black balls)=3 × P(White balls)

$$\Rightarrow \quad \frac{x}{15+x} = 3\times\frac{15}{15+x}$$
$$\Rightarrow \quad x = 45$$

Hence, number of black balls in the bag is 45.

Question 15. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, –2) and Q(3, 7)? Also find the value of y.

Solution.

Let point R$\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, –2) and Q(3, 7) in ratio k:1.

Thus using section formula, $x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$, we get the coordinates of R$\left(\frac{24}{11}, y\right)$ as:

$$\frac{24}{11} = \frac{3k+2}{k+1}, y = \frac{7k-2}{k+1}$$
$$\Rightarrow \quad 24(k+1) = 11(3k+2),\ y(k+1) = 7k-2$$
$$\Rightarrow \quad 24k + 24 = 33k + 22,\ yk + y = 7k - 2$$
$$\Rightarrow \quad 9k = 2$$
$$\Rightarrow \quad k = \frac{2}{9}$$

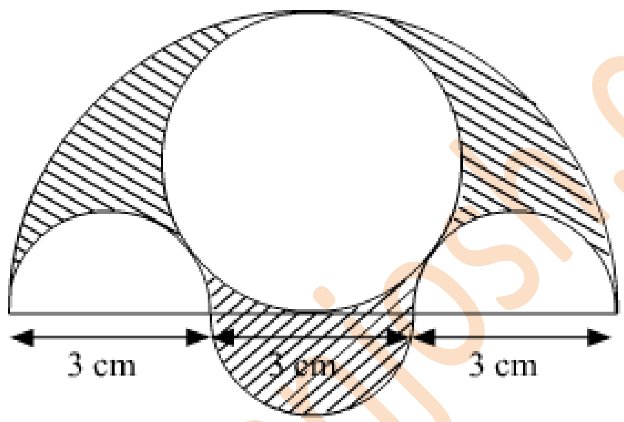
Putting $k = \frac{2}{9}$ in $yk + y = 7k - 2$, we get :

$$y\left(\frac{2}{9}\right) + y = 7\left(\frac{2}{9}\right) - 2$$
$$\Rightarrow \quad \frac{2y+9y}{9} = \frac{14-18}{9}$$
$$\Rightarrow \quad \frac{11y}{9} = \frac{-4}{9}$$
$$\Rightarrow \quad y = \frac{-4}{11}$$

Thus the given point $\left(\frac{24}{11}, y\right)$ divides the the line segment in k:1 = $\frac{9}{2}:1 \text{ or } 9:2$.

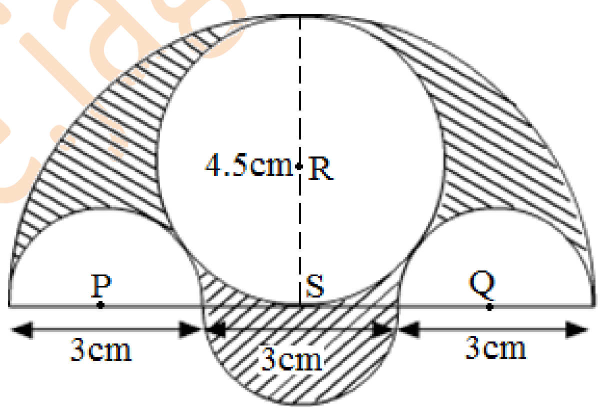
And value of y is $\frac{-4}{11}$.

Question 16. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



Solution.

Given diagram can be represented as below:



Here, area of shaded region = Area of largest semicircle – Area of circle with centre R – Sum of areas two of semicircles with cemtres as P and Q respectively + Area of semicircle with centre S.

$$= \frac{\pi \times 4.5 \times 4.5}{2} - \pi \times \frac{4.5}{2} \times \frac{4.5}{2} - 2\frac{\pi \times 3 \times 3}{2 \times 2 \times 2} + \frac{\pi \times 3 \times 3}{2 \times 2 \times 2}$$

$$= \pi \times \frac{(4.5)^2}{2}\left[1 - \frac{1}{2}\right] - \frac{\pi \times 3 \times 3}{2 \times 2 \times 2}$$

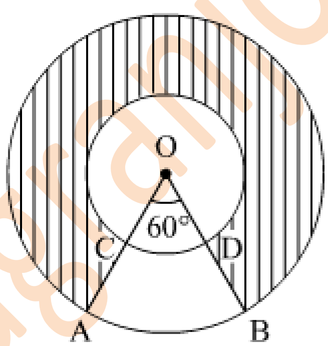
$$= \pi \times \frac{20.25}{4} - \pi \times \frac{9}{8}$$

$$= \pi\left(5.06 - 1.12\right)$$

$$= \frac{22}{7} \times 3.94 = 12.38\,\text{cm}^2$$

Thus, the required area of shaded region = 12.38cm^2.

Question 17. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If ∠AOB = 60°, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



Solution.

Given: Radius of the inner circle, r = 21 cm
And, radius of the outer circle, R = 42 cm
Also, ∠AOB = 60°
Now, area of the shaded region = Area of outer circle – Area of inner circle – Area of ABCD

$$= \pi R^2 - \pi r^2 - \left[\frac{60}{360} \times \pi \times R^2 - \frac{60}{360} \times \pi \times r^2\right]$$
$$= \pi\left(R^2 - r^2\right) - \frac{60}{360} \times \pi\left(R^2 - r^2\right)$$
$$= \pi\left(R^2 - r^2\right)\left[1 - \frac{60}{360}\right]$$
$$= \frac{22}{7}\left(42^2 - 21^2\right)\left(\frac{5}{6}\right)$$
$$= \frac{22}{7}(42 + 21)(42 - 21) \times \frac{5}{6}$$
$$= \frac{22}{7} \times 63 \times 21 \times \frac{5}{6}$$
$$= 3465\,\text{cm}^2$$

Thus, the required area of shaded region = 3465 cm^2.

Question 18. Water in a canal, 5·4 m wide and 1·8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

Solution.

Given, width of the canal = 5.4 m
Depth of the canal = 1.8 m

And, speed of the flowing water = 25 km/h = $\frac{25000}{60} = \frac{1250}{3}$ m/min

Thus, volume of water flowing out of the canal in 1 min = Area of opening of canal × $\frac{1250}{3}$

= 5.4 × 1.8 × $\frac{1250}{3}$

= 4050 m^3

⇒Volume of water flowing out of the canal in 40 min = 40 × 4050 m^3 = 162000 m^3

Now, height of the standing water required for irrigation = 10 cm = 0.1 m

So, Area of irrigation = $\dfrac{\text{Volume of water flowing out of canal in 40 minutes}}{\text{Height of standing water required for irrigation}}$

$$= \frac{162000}{0.1} = 1620000\,\text{m}^2$$

$= 162$ hectare $\left(\because 1\,\text{hectare} = 10000\,\text{m}^2\right)$

Therefore, total area that can be irriadated by the canal in 40 minutes = 162 m^2.

Question 19. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution.

Given, perimeter of upper end, C_1 = 18cm,

And, perimeter of lower end, C_2 = 6 cm and

Slant Height, l = 4 cm

Let the radius of upper end be R_1 and the radius of lower end be R_2.

Therefore, C_1 = 18 cm

$\Rightarrow \quad 2\pi R_1 = 18$

$\Rightarrow \quad R_1 = \dfrac{18}{2\pi}$

Also, C_2 = 6

$\Rightarrow \quad 2\pi R_2 = 6$

$\Rightarrow \quad R_2 = \dfrac{6}{2\pi}$

Now, the curved surface area of the frustum of a cone = $\pi\left(R+r\right)l$

$$= \pi\left(\frac{18}{2\pi} + \frac{6}{2\pi}\right)\times 4$$
$$= \left(9+3\right)\times 4$$
$$= 48\,\text{cm}^2$$

Thus, the required surface area of given frustum = 48 cm^2.

Question 20. The dimensions of a solid iron cuboid are 4·4 m × 2·6 m × 1·0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

Solution.

Given, dimensions of a solid iron cuboid = 4·4 m × 2·6 m × 1·0 m

Thus, Voume of cuboid = 11.44 m^3

Also given, inner radius of cylindrical pipe, r = 30 cm = 0.30 m

And, thickness of cylindrical pipe = 5 cm

$\therefore$ Outer radius of cylindrical pipe, R = 30 + 5 = 35 cm = 0.35 m

Let the length of the cylindrical pipe be h cm.

Therefore, volume of cylindrical pipe $= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

$$= \pi h\,[(0.35)^2 - (0.30)^2]$$
$$= \pi h(0.35 - 0.30)(0.35 + 0.30)$$
$$= \pi h(0.05 \times 0.65) = 0.0325\pi h \text{ m}^3$$

According to the question,

Volume of iron cylindrical pipe = Volume solid iron cuboid

$$\Rightarrow \quad 0.0325\pi h = 11.44$$

$$\Rightarrow \quad h = \frac{1144 \times 100 \times 7}{325 \times 22} = 112\,\text{m}$$

Thus, the required length of pipe = 112 m.

SECTION D

Question 21. Solve for x:

$$\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$$

Solution.

Given: $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$

$$\Rightarrow \frac{5x+1+3x+3}{(x+1)(5x+1)} = \frac{5}{x+4}$$
$$\Rightarrow (8x+4)(x+4) = 5(x+1)(5x+1)$$
$$\Rightarrow 8x^2+32x+4x+16 = 5(5x^2+x+5x+1)$$
$$\Rightarrow 8x^2+36x+16 = 25x^2+30x+5$$
$$\Rightarrow 17x^2-6x-11=0$$
$$\Rightarrow 17x^2-17x+11x-11=0 \quad [\text{Splitting the middle term}]$$
$$\Rightarrow 17x(x-1)+11(x-1)=0$$
$$\Rightarrow (17x+11)(x-1)=0$$
$$\Rightarrow 17x+11=0 \text{ or } x-1=0$$
$$\Rightarrow x=\frac{-11}{17} \text{ or } x=1$$

Question 22. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

Solution.

Let time taken by one tap to fill the tank = x hours

Then, time taken by second tap to fill the tank = x + 3 hours

Thus, part of the tank filled by first tap in 1 hour = $\frac{1}{x}$

And, part of the tank filled by second tap in 1 hour = $\frac{1}{x+3}$

Thus, part of the tank filled by both the taps in 1 hour = $\frac{1}{x}+\frac{1}{x+3}$ …..(i)

But, given that time taken by both the taps to fill the tank = $3\frac{1}{13}=\frac{40}{13}$h

$$\frac{1}{x}+\frac{1}{x+3}=\frac{13}{40}$$

$$\Rightarrow \quad \frac{x+3+x}{x(x+3)}=\frac{13}{40}$$

$$\Rightarrow \quad 40(2x+3)=13(x^2+3x)$$

$$\Rightarrow \quad 80x+120=13x^2+39x$$

$$\Rightarrow \quad 13x^2-41x-120=0$$

$$\Rightarrow \quad 13x^2-65x+24x-120=0$$

$$\Rightarrow \quad 13x(x-5)+24(x-5)=0$$

$$\Rightarrow \quad (13x+24)(x-5)=0$$

$$\Rightarrow \quad 13x+24=0 \text{ or } x-5=0$$

$$\Rightarrow \quad x=\frac{-24}{13} \text{ or } x=5$$

i.e., in $\frac{40}{13}$ h , part of tank filled by both taps = 1

$\Rightarrow$ In 1 h, part of tank filled by both taps = $\frac{13}{40}$ …..(ii)

From equations (i) and (ii), we have:

$$\frac{1}{x}+\frac{1}{x+3}=\frac{13}{40}$$

$$\Rightarrow \quad \frac{x+3+x}{x(x+3)}=\frac{13}{40}$$

$$\Rightarrow \quad 40(2x+3)=13(x^2+3x)$$

$$\Rightarrow \quad 80x+120=13x^2+39x$$

$$\Rightarrow \quad 13x^2-41x-120=0$$

$$\Rightarrow \quad 13x^2-65x+24x-120=0$$

$$\Rightarrow \quad 13x(x-5)+24(x-5)=0$$

$$\Rightarrow \quad (13x+24)(x-5)=0$$

$$\Rightarrow \quad 13x+24=0 \text{ or } x-5=0$$

$$\Rightarrow \quad x=\frac{-24}{13} \text{ or } x=5$$

But time cannot be negative.

Therefore, taking x = 5, we get:

Time taken by one tap to fill the tank = 5 hours

And time taken by the second tap to fill the tank = 5 + 3 = 8 hours.

Question 23. If the ratio of the sum of the first n terms of two A.Ps is (7n + 1) : (4n + 27), then find the ratio of their 9th terms.

Solution.

Let a_1, a_2 be the first terms of the given two A.Ps and d_1, d_2 be their respective common differences.
Then the sums of the n terms of first A.P. is given as:

$$S_n = \frac{n}{2}\left\{2a_1 + (n-1)d_1\right\} \qquad ....(\text{i})$$

Then the sums of the n terms of second A.P. is given as:

$$S_n' = \frac{n}{2}\left\{2a_2 + (n-1)d_2\right\} \qquad ....(\text{ii})$$

According to the question

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

Using equations (i) and (ii), we get:

$$\frac{\frac{n}{2}\left\{2a_1 + (n-1)d_1\right\}}{\frac{n}{2}\left\{2a_2 + (n-1)d_2\right\}} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{\left\{2a_1 + (n-1)d_1\right\}}{\left\{2a_2 + (n-1)d_2\right\}} = \frac{7n+1}{4n+27} \qquad ....(\text{iii})$$

Let's the mth terms of the two AP's be a_m and a'_m
As the nth term of an AP is given as,

$a_n = a + (n-1)d$

Thus the ratio of the mth terms of the two AP's is given as:

$$\therefore \quad \frac{a_m}{a'_m} = \frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2}$$

Multiplying the numerator and denominator of R.H.S by two we get:

$$\frac{a_m}{a'_m} = \frac{2a_1 + 2(m-1)d_1}{2a_2 + 2(m-1)d_2}$$

$$\Rightarrow \quad \frac{a_m}{a'_m} = \frac{2a_1 + [(2m-1)-1]d_1}{2a_2 + [(2m-1)-1]d_2}$$

$$\Rightarrow \quad \frac{a_m}{a'_m} = \frac{S_{2m-1}}{S'_{2m-1}}$$

$$\Rightarrow \quad \frac{a_m}{a'_m} = \frac{7(2m-1)+1}{4(2m-1)+27} \quad [\text{Using (iii)}]$$

$$\Rightarrow \quad \frac{a_m}{a'_m} = \frac{14m-7+1}{8m-4+27}$$

$$\Rightarrow \quad \frac{a_m}{a'_m} = \frac{14m-6}{8m+23}$$

Substituting $'m'$ by 9 in above equation, we get:

$$\frac{a_9}{a'_9} = \frac{14(9)-6}{8(9)+23}$$

$$\Rightarrow \quad \frac{a_9}{a'_9} = \frac{120}{95}$$

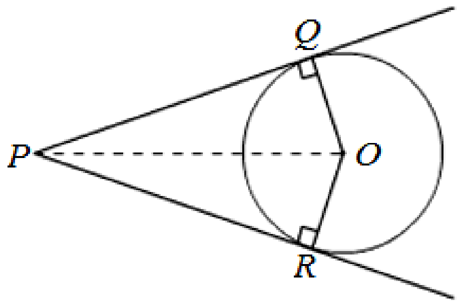
Hence, the ratio the 9^{th} terms of the two APs = 120:95 or 24:19.

Question 24. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Solution.

Given: PQ and PR are two tangents drawn from an external point P to the a circle with centre O and radius R.
To prove: PR = PQ

Construction: Join OP.
Proof:



We know that a tangent to the circle is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OQP = \angle ORP = 90°$

In Δ OQP and ΔORP,

OP = OP (Common)

OQ = OR (Radius of the circle)

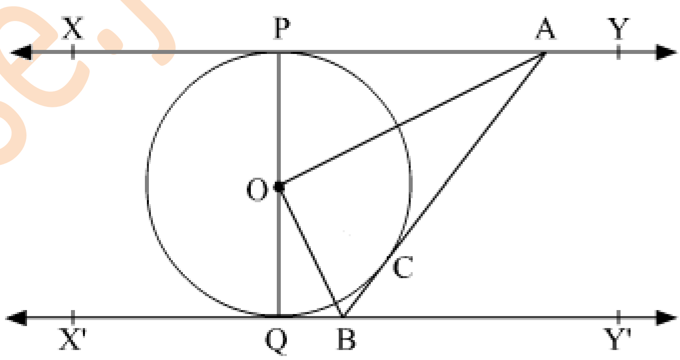
$\angle$ OQP = $\angle$ ORP = 90°

$\therefore$ Δ OQP $\cong$ ΔORP (RHS congruence criterion)

$\Rightarrow$ PQ = PR (CPCT)

Hence, the lengths of the tangents drawn from an external point to a circle are equal.

Question 25. In the given figure, XY and X'Y' are two parallel tangents to circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90°$.



Solution.

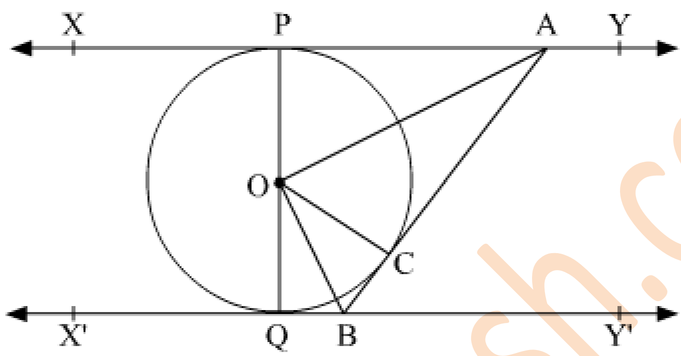
Given: XY and X′Y′ are two parallel tangents to the circle with centre O.

And AB is another tangent at the point C, which intersects XY at A and X′Y′ at B.

To prove: ∠AOB = 90°

Construction: Join OC.

Proof:



In ΔOAP and ΔOAC,

OP = OC (Radii of the same circle)

OA = OA (Common side)

AP = AC (Length of tangents drawn from an external point to a circle are equal)

ΔOAP ≅ ΔOAC (SSS congruence criterion)

∴ ∠AOP = ∠COA (C.P.C.T) .....(i)

Similarly, ΔOBQ ≅ ΔOBC

∴ ∠BOQ = ∠COB .....(ii)

POQ is a diameter of the circle. Hence, it is a straight line.

∴ ∠AOP + ∠COA + ∠BOQ + ∠COB = 180º

2∠COA + 2∠COB = 180º [Using (i) and (ii)]

⇒ ∠COA + ∠COB = 90º

⇒ ∠AOB = 90°

Hence proved.

Question 26. Construct a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC.

Solution.

Here in ΔABC, $\angle B = 45°$ and $\angle A = 105°$

Now, $\angle A + \angle B + \angle C = 180^o$ [Sum of interior angles of a triangle is 180^o]

$\Rightarrow \quad 105° + 45° + \angle C = 180^o$

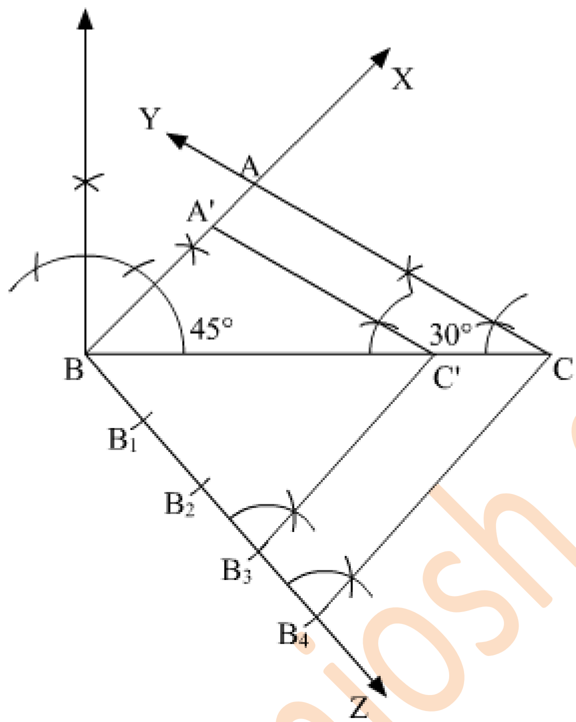
$\Rightarrow \quad \angle C = 180° - (105° + 45°) = 30°$

Now we draw the ΔABC by following the steps of construction as given below:

1. Draw line BC = 7 cm.
2. At B, construct $\angle CBX = 45°$ and at C, construct $\angle BCY = 30°$.
3. The point of intersection of BX and CY gives A. Thus, ΔABC is obtained.

Now to draw another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC, follow the steps given below:

4. Draw any ray BZ making an acute angle with BC on the side opposite to the vertex A.
5. Locate four points B_1, B_2, B_3 and B_4 on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
6. Join B_4C and draw a line through B_3 parallel to B_4C to intersect BC at C'.
7. Draw a line through C' parallel to the line CA to intersect BA at A'.

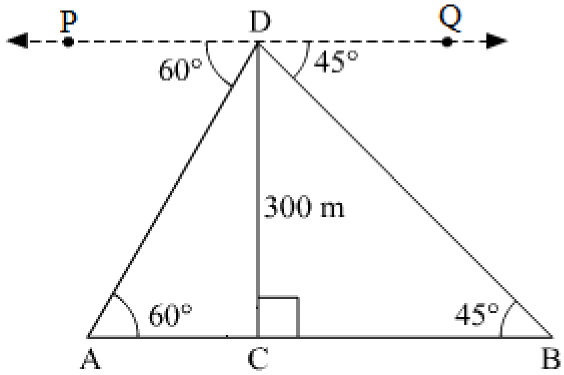


Thus, $\Delta A'BC'$ is the required triangle similar to the ΔABC with scale factor as $\frac{3}{4}$.

Question 27. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1.732$]

Solution.

Let D be the point of location of aeroplane lying 300 height m above the ground from where it makes angles of depression as 45° and 60° of the two banks say A and B of a river.



Here, $\angle CAD = \angle PDA = 60^\circ$ (Alternate angles)

$\angle CBD = \angle QDB = 45^\circ$ (Alternate angles)

Now, in right ΔACD,

$$\tan 60^\circ = \frac{CD}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{300}{AC}$$

$$\Rightarrow \quad AC = \frac{300}{\sqrt{3}}$$

$$\Rightarrow \quad AC = \frac{300}{3}\sqrt{3} = 100\sqrt{3}$$

$$\Rightarrow \quad AC = 100 \times 1.732 = 173.2\,\text{m}$$

Also, in right ΔBCD,

$$\tan 45^\circ = \frac{CD}{BC}$$

$$\Rightarrow \quad 1 = \frac{300}{BC}$$

$$\Rightarrow \quad BC = 300\text{m}$$

$\therefore$ Width of the river, AB = AC + BC = $173.20 + 300 = 473.20\,\text{m}$

Thus, the width of the river is 473.2 m.

Question 28. If the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k − 1, 5k) are collinear, then find the value of k.

Solution.

Given that points A(k + 1, 2k), B(3k, 2k + 3) and C(5k − 1, 5k) are collinear.

$\therefore$ The area of the triangle formed by these three points = 0

$$\Rightarrow \frac{1}{2}\left|x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right|=0$$
$$\Rightarrow \frac{1}{2}\left|(k+1)\left[(2k+3)-(5k)\right]+(3k)\left\{(5k)-(2k)\right\}+(5k-1)\left[(2k)-(2k+3)\right]\right|=0$$
$$\Rightarrow (k+1)[-3k+3]+(3k)[3k]+(5k-1)[-3]=0$$
$$\Rightarrow -3k^2+3k-3k+3+9k^2-15k+3=0$$
$$\Rightarrow 6k^2-15k+6=0$$

Dividing by 3 on both sides of above equation,we get:

$$2k^2-5k+2=0$$
$$\Rightarrow 2k^2-4k-k+2=0 \qquad \left[\text{Splitting the middle term}\right]$$
$$\Rightarrow 2k(k-2)-1(k-2)=0$$
$$\Rightarrow (2k-1)(k-2)=0$$
$$\Rightarrow 2k-1=0 \text{ or } k-2=0$$
$$\Rightarrow k=\frac{1}{2},2$$

Thus the required values of k are $\frac{1}{2}$ and 2.

Question 29. Two different dice are thrown together. Find the probability that the numbers obtained have
(i) even sum, and
(ii) even product.

Solution.

On throwing the two different dice the various outcomes obtained are as follows:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2),(2,3) (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Thus, here total number of outcomes are 36.
(i) Even sum outcomes are as follows:

(1,1), (1,3), (1,5),
(2,2), (2,4), (2,6),
(3,1), (3,3), (3,5),
(4,2), (4,4), (4,6),
(5,1), (5,3), (5,5),
(6,2), (6,4), (6,6)

Thus, number of favourable outcomes = 18

$$P(\text{even sum}) = \frac{18}{36} = \frac{1}{2}$$

Hence, the probability of getting an even sum is $\frac{1}{2}$.

(ii) Even product outcomes are as follows:

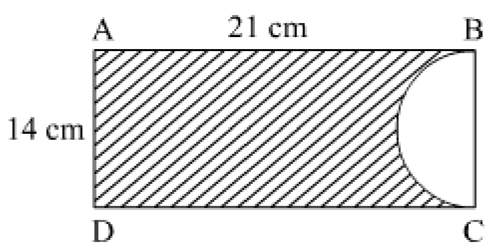
(1,2), (1,4), (1,6),
(2,1), (2,2),(2,3) (2,4), (2,5), (2,6),
(3,2), (3,4), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,2), (5,4), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Thus number of favourable outcomes = 27

$$P(\text{even product}) = \frac{27}{36} = \frac{3}{4}$$

Hence, the probability of getting an even product is $\frac{3}{4}$.

Question 30. In the given figure, ABCD is rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Solution.

Given, dimensions of rectangle ABCD are 21 cm × 14 cm.

Also diameter of semicircle = CB = 14 cm

$\therefore$ Radius of semicircle, r = $\frac{14}{2} = 7\,\text{cm}$

Now, Area of shaded region = Area of rectangle ABCD – Area of semicircle with radius 7cm

$$= (\text{Length} \times \text{Breadth}) - \frac{1}{2}\pi r^2$$
$$= (21 \times 14) - \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right)$$
$$= 294 - 77 = 217\text{cm}^2$$

Thus, required area of shaded region = 217 cm^2

Now, perimetre of shaded region = AB + AD + DC + $\overset{\frown}{BC}$

Now, perimeter of Arc BC = $\overset{\frown}{BC}$ = $\pi r = \frac{22}{7} \times 7 = 22\,\text{cm}$

Thus, perimetre of shaded region = 21 + 14 + 21 + 22 = 78 cm

Question 31. In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3·5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation.

Solution.

Given:

Length of the roof, l = 22m

Width of the roof, b = 20 m

Radius of the cylindrical vessel, $r = \frac{2}{2} = 1\text{m}$

And height of the cylindrical vessel, h = 3.5 m

Let the height of the rainfall be H m.

Now, Volume of water on roof = $l \times b \times H = 22 \times 20 \times H \text{ m}^3$

Also, total amount of rainfall = Volume of rain water collected in the cylindrical vessel

$\Rightarrow \quad 22 \times 20 \times H = \pi r^2 h$

$\Rightarrow \quad 440H = \frac{22}{7} \times 1^2 \times 3.5$

$\Rightarrow \quad H = \frac{22 \times 0.5}{440}$

$\Rightarrow \quad H = 0.025 \text{ m} = 2.5 \text{ cm}$

Water conservation is a way to improve the conditions of scarcity of water and ensures the availability of water throughout the year. Rain water harvesting is one of the most important. Harvested and filtered rain water could be used for toilets, home gardening and small scale agriculture. People should avoid the contamination of water by not washing clothes, bathing their cattles, and throwing garbages in the river or pond water.