

Solved Question Paper 2018
CBSE Class 10
Subject – Mathematics

Time allowed: 3 hours Maximum

Marks: 80

General Instructions:

- i. All questions are compulsory.
- ii. This question paper consists of 30 questions divided into four sections – A, B, C and D.
- iii. Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
- iv. There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
- v. Use of calculator is not permitted.

SECTION – A

1. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k .

Solution. $x = 3$ is one root of the equation

$$\therefore 9 - 6k - 6 = 0$$

$$\Rightarrow k = \frac{1}{2}$$

2. What is the HCF of smallest prime number and the smallest composite number?

Solution. The required number 2 and 4.

HCF of 2 and 4 is 2.

3. Find the distance of a point $P(x, y)$ from the origin.

Solution.

$$OP = \sqrt{x^2 + y^2}$$

4. In an AP, if the common difference (d) = -4, and the seventh term (a_7) is 4, then find the first term.

Solution.

$$a + 6(-4) = 4$$

$$\Rightarrow a = 28$$

5. What is the value of $(\cos^2 67^\circ - \sin^2 23^\circ)$?

Solution.

$$\because \cos^2 67^\circ = \sin^2 23^\circ$$

$$\therefore \cos^2 67^\circ - \sin^2 23^\circ = 0$$

6. Given $\triangle ABC \sim \triangle PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\triangle ABC}{ar\triangle PQR}$.

Solution.

$$\frac{ar\triangle ABC}{ar\triangle PQR} = \frac{AB^2}{PQ^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

SECTION – B

7. Given $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

Solution.

Let us assume that $(5 + 3\sqrt{2})$ is a rational number.

$$\therefore 5 + 3\sqrt{2} = \frac{p}{q} \text{ where } q \neq 0 \text{ and } p \text{ and } q \text{ are integers.}$$

$$\Rightarrow \sqrt{2} = \frac{p - 5q}{3q}$$

$$\Rightarrow \sqrt{2} \text{ is a rational number as RHS is rational.}$$

This contradicts the given fact that $\sqrt{2}$ is irrational.

Hence, $(5 + 3\sqrt{2})$ is an irrational number.

8. Fig. 1, ABCD is a rectangle. Find the values of x and y .

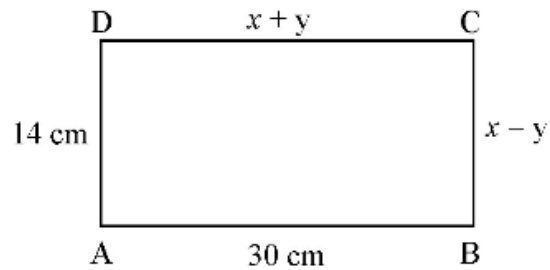


Fig. – 1

Solution. $AB = DC$ and $BC = AD$

$$\Rightarrow x + y = 30 \quad \dots(i)$$

$$\text{And } x - y = 14 \quad \dots(ii)$$

Adding (i) and (ii), we get:

$$2x = 24$$

$$\Rightarrow x = 12$$

Substituting the above value of x in equation (i), we get:

$$22 + y = 30$$

$$\Rightarrow y = 8.$$

9. Find the sum of first 8 multiples of 3.

Solution.

$$S = 3 + 6 + 9 + 12 + \dots + 24$$

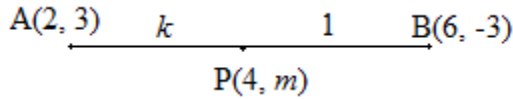
$$= 3(1 + 2 + 3 + \dots + 8)$$

$$= 3 \times \frac{8 \times 9}{2}$$

$$= 108$$

10. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, 3). Hence find m.

Solution. Let AP : PB = k : 1



$$\therefore \frac{6k+2}{k+1} = 4$$

$$\Rightarrow k = 1, \text{ ratio is } 1:1$$

$$\text{Hence, } m = \frac{-3+3}{2} = 0$$

11. Two different dice are tossed together. Find the probability:

i. of getting a doublet

ii. of getting a sum 10, of the numbers on the two dice.

Solution.

Total number of possible outcomes = 36

i. Doublets are (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)

Total number of doublets = 6

$$\therefore \text{Prob (getting a doublet)} = \frac{6}{36} = \frac{1}{6}$$

ii. Favourable outcomes are (4, 6) (5, 5) (6, 4) i.e., 3

$$\therefore \text{Prob (getting a sum 10)} = \frac{3}{36} = \frac{1}{12}$$

12. An integer is chosen at random between 1 and 100. Find the probability that it is : i. divisible by 8. ii. not divisible by 8.

Solution. Total number of outcomes = 98

i. Favourable outcomes are 8, 16, 24, ..., 96 i.e., 12

$$\therefore \text{Prob (integer is divisible by 8)} = \frac{12}{98} = \frac{6}{49}$$

ii. Prob (integer is not divisible by 8) = $1 - \frac{6}{49} = \frac{43}{49}$

SECTION – C

13. Find HCF and LCM of 404 and 96 and verify that $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$.

Solution.

Here, $404 = 2 \times 2 \times 101$

And $96 = 2 \times 2 \times 2 \times 2 \times 3$

Common factors = 2×2

$\therefore \text{HCF of } 404 \text{ and } 96 = 2 \times 2 = 4$

$\text{LCM of } 404 \text{ and } 96 = 2 \times 2 \times 2 \times 2 \times 3 \times 101 = 9696$

Now, $\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$

And product of the two numbers = $404 \times 96 = 38784$

Hence, it is verified. $\text{HCF} \times \text{LCM} = \text{Product of } 404 \text{ and } 96$.

14. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are

$(2 + \sqrt{3})$ and $(2 - \sqrt{3})$.

Solution.

Let $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

Given, $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ are zeroes of $p(x)$

$$\begin{aligned}\therefore p(x) &= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) \times g(x) \\ &= (x^2 - 4x + 1)g(x)\end{aligned}$$

$$\Rightarrow g(x) = p(x)/(x^2 - 4x + 1)$$

$$\Rightarrow g(x) = (2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1)$$

$$\Rightarrow g(x) = 2x^2 - x - 1 = (2x + 1)(x - 1)$$

Therefore other zeroes of $p(x)$ are $x = -\frac{1}{2}$ and $x = 1$.

Thus, all the zeroes of $p(x)$ are $(2 + \sqrt{3}), (2 - \sqrt{3}), -\frac{1}{2}$ and 1 .

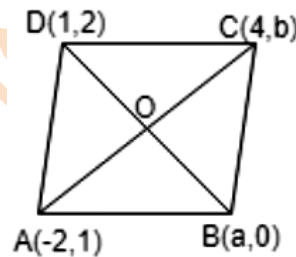
15. If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram ABCD, find the values of a and b . Hence find the lengths of its sides.

OR

If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

Solution.

Here, $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram ABCD.



As ABCD is a parallelogram.

$\therefore$ Diagonals AC and BD bisect each other.

$\Rightarrow$ Mid point of BD is same as that of AC.

$$\Rightarrow \left(\frac{a+1}{2}, \frac{2}{2} \right) = \left(\frac{-2+4}{2}, \frac{b+1}{2} \right)$$

$$\Rightarrow \left(\frac{a+1}{2}, 1 \right) = \left(1, \frac{b+1}{2} \right)$$

$$\Rightarrow \frac{a+1}{2} = 1 \text{ and } \frac{b+1}{2} = 1$$

$$\Rightarrow a = 1 \text{ and } b = 1$$

Using distance formula we can find out the length of the sides of parallelogram.

$$\text{Distance formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(1+2)^2 + (0-1)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10}u$$

$$BC = \sqrt{(4-1)^2 + (1-0)^2}$$

$$= \sqrt{9+1}$$

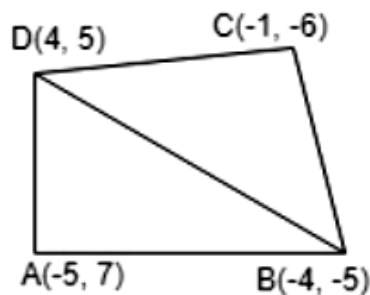
$$= \sqrt{10}u$$

Thus, the lengths of all four sides of parallelogram are $\sqrt{10}u$.

OR

Solution.

Given that A(-5, 7), B(-4, -5), C(-1, -6) and D(4, 5) are the vertices of a quadrilateral ABCD.



$$\text{Area of quad ABCD} = \text{Ar}\triangle ABD + \text{Ar}\triangle BCD$$

$$\text{Area of } \triangle ABD = \frac{1}{2} |(-5)(-5-5) + (-4)(5-7) + (4)(7+5)|$$

$$= 53 \text{ sq units}$$

$$\text{Area of } \triangle BCD = \frac{1}{2} |(-4)(-6-5) + (-1)(5+5) + (4)(-5+6)|$$

$$= 19 \text{ sq units}$$

Hence area of quadrilateral ABCD = 53 + 19 = 72 sq units.

16. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

Solution.

Let the usual speed of the plane be x km/hr

Distance to the destination = 1500 km

Time taken to reach the destination at usual speed,

$$t_1 = \frac{1500}{x} \text{ hr}$$

Increased speed of the plane = $(x + 100)$ km/hr

Time taken to reach the destination with increased speed,

$$t_2 = \frac{1500}{x+100} \text{ hr}$$

Given that, $t_1 - t_2 = 30 \text{ min}$

$$\begin{aligned}\frac{1500}{x} - \frac{1500}{x+100} \text{ hr} &= \frac{30}{60} \text{ hr} \\ \Rightarrow \frac{1500x + 150000 - 1500x}{x(x+100)} &= \frac{1}{2} \\ \Rightarrow \frac{150000}{x^2 + 100x} &= \frac{1}{2} \\ \Rightarrow x^2 + 100x - 300000 &= 0 \\ \Rightarrow x^2 + 600x - 500x - 300000 &= 0 \\ \Rightarrow (x + 600)(x - 500) &= 0 \\ \Rightarrow x \neq -600 \\ \therefore x &= 500\end{aligned}$$

Therefore, usual speed of plane = 500km/hr.

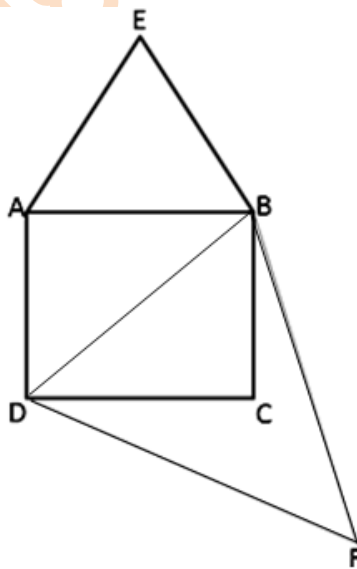
17. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

OR

If the areas of two similar triangles are equal, prove that they are congruent.

Solution.

Let ABCD be a square of side a.



Two desired equilateral triangles are formed as ABE and DBF.

Here $BD^2 = BC^2 + CD^2$

$$\Rightarrow BD^2 = a^2 + a^2 = 2a^2$$

$$\Rightarrow BD = \sqrt{2}a$$

Thus, Length of one side of $\triangle DBF = \sqrt{2}a$ units

$$\text{Area of equilateral } \triangle DBF = \frac{\sqrt{3}}{4}(\sqrt{2}a)^2 = \frac{\sqrt{3}}{2}a^2 \text{ sq.unit}$$

Also, length of one side of $\triangle ABE = a$ units

$$\text{Area of equilateral } \triangle ABE = \frac{\sqrt{3}}{4}(a)^2 = \frac{\sqrt{3}}{4}a^2 \text{ sq.unit}$$

$$\text{Thus, ar}(\triangle ABE) = \frac{1}{2} \text{ ar}(\triangle DBF)$$

OR

Solution.

Let $\triangle ABC$ and $\triangle PQR$ be the two similar triangles with

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle PQR)$$

For similar triangles we have:

$$\frac{\text{ar}\triangle ABC}{\text{ar}\triangle PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

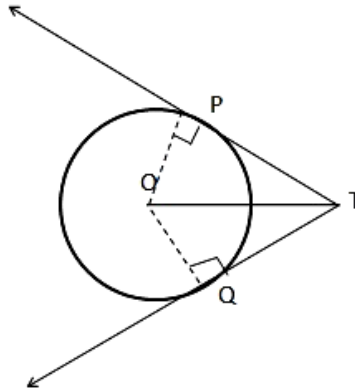
$$\Rightarrow 1 = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

$$\Rightarrow AB = PQ, BC = QR, AC = PR$$

Therefore, $\triangle ABC \cong \triangle PQR$ (by SSS congruence rule)

18. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution.



To prove: $PT = QT$

Proof: Consider the triangles OPT and OQT.

$$OP = OQ$$

$$\angle OPT = \angle OQT = 90^\circ$$

$$OT = OT \text{ (common side)}$$

$$\Rightarrow OPT \cong OQT \text{ (by RHS congruence rule)}$$

$$\text{Hence } PT = QT \quad (\text{CPCT})$$

Hence Proved.

19. If $4 \tan \theta = 3$, evaluate $\left(\frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} \right)$.

OR

If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A .

Solution.

Given, $4 \tan \theta = 3$,

$$\Rightarrow \tan \theta = \frac{3}{4}$$

$$\Rightarrow \tan \theta = \frac{3}{4} \left(\frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} \right)$$

$$\Rightarrow \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

$$\Rightarrow \text{Hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\therefore \sin \theta = \frac{3}{5} \text{ and } \cos \theta = \frac{4}{5}$$

$$\begin{aligned} \therefore \frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} &= \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1} \\ &= \frac{13}{11} \end{aligned}$$

OR

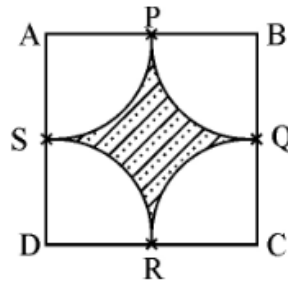
$$\tan 2A = \cot(A - 18^\circ)$$

$$\Rightarrow 90^\circ - 2A = A - 18^\circ$$

$$\Rightarrow 3A = 108^\circ$$

$$\Rightarrow A = 36^\circ$$

20. Find the area of the shaded region in Fig. 2, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. [Use = 3.14].

**Fig. – 2****Solution.**

Area of the shaded region = Area of the square ABCD – Area of 4 quadrants

Here, radius of each arc drawn = $12/2 = 6\text{ cm}$

$$\text{Area of one quadrant} = \frac{\pi r^2}{4} \text{ cm}^2 = 3.14 \times \frac{36}{4} \text{ cm}^2$$

$$\text{Area of four quadrants} = 4 \times 3.14 \times \frac{36}{4} = 113.04 \text{ cm}^2$$

$$\text{Area of square ABCD} = 12 \times 12 = 144 \text{ cm}^2$$

$$\text{Hence Area of shaded region} = 144 - 113.04 = 30.96 \text{ cm}^2$$

21. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 3. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.

**Fig. 3****OR**

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Solution.

Total surface Area of article = curved surface area of cylinder + 2(curved surface area of a hemisphere)

$$\begin{aligned}\text{Now, curved surface area of cylinder} &= 2\pi rh = 2 \times \frac{22}{7} \times 3.5 \times 10 \\ &= 220\text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Curved surface area of two hemispherical scoops} &= 2(2\pi r^2) \\ &= 4 \times \frac{22}{7} \times 3.5 \times 3.5 \\ &= 88 \times 0.5 \times 3.5 = 154\text{cm}^2\end{aligned}$$

$$\text{Total surface Area of article} = 220 + 154 = 374 \text{ cm}^2$$

OR

Solution. Radius of conical heap = 12m

$$\begin{aligned}\text{Volume of rice} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \\ &= 528\text{m}^3\end{aligned}$$

Area of canvas cloth required to just cover the heap of rice = πrl

$$\text{Here, slant height, } l = \sqrt{(12)^2 + (3.5)^2} = 12.5\text{m}$$

$$\text{Thus, Area of canvas required} = \frac{22}{7} \times 12 \times 12.5 = 471.4\text{m}^2$$

22. The table below shows the salaries of 280 persons:

Salary (In thousand Rs)	No. of persons
5-10	49

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10-15	133
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4
40-45	2
45-50	1

Calculate the median salary of the data.

Solution.

Salary (In thousand Rs)	No. of persons	cf
5-10	49	49
10-15	133	182
15-20	63	245
20-25	15	260
25-30	6	266
30-35	7	273
35-40	4	277
40-45	2	279
45-50	1	280

$$\frac{N}{2} = \frac{280}{2} = 140$$

Median class is 10-15

$$\text{Median} = l + \frac{h}{f} \left(\frac{N}{2} - C \right)$$

$$= 10 + \frac{5}{133} (140 - 49)$$

$$= 10 + \frac{5 \times 91}{133}$$

$$= 13.42$$

Median salary is Rs 13.42 thousand or Rs 13420 (approx)

SECTION - D

23. A motor boat whose speed is 18 km/hr in still water takes 11r more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. if it takes 3 hours to complete total journey, what is the original average speed?

Solution. Let the speed of stream be x km/hr.

$$\therefore \left. \begin{array}{l} \text{The speed of the boat upstream} = (18 - x) \text{ km/hr} \\ \text{and speed of the boat downstream} = (18 + x) \text{ km/hr} \end{array} \right\}$$

As given in the question,

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow (x + 54)(x - 6) = 0$$

$$x \neq -54, \therefore x = 6$$

$\therefore$ speed of the stream = 6 km/hr.

OR

Let the original average speed of train be x km/hr.

$$\text{Therefore } \frac{63}{z} + \frac{72}{x+6} = 3$$

$$= x^2 - 39x - 126 = 0$$

$$= (x - 42)(x + 3) = 0$$

$$x \neq -3 \therefore x = 42$$

Original speed of train is 42 km/hr.

24. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

Solution. Let the four consecutive terms of the A.P. be

$$a - 3d, a - d, a + d, a + 3d.$$

By given conditions

$$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$$

$$\Rightarrow 4a = 32$$

$$\Rightarrow a = 8$$

$$\text{and } \frac{(a - 3d)(a + 3d)}{(a - d)(a + d)} = \frac{7}{15}$$

$$\Rightarrow 8a^2 = 128d^2$$

$$\Rightarrow d^2 = 4$$

$$\Rightarrow d = \pm 2$$

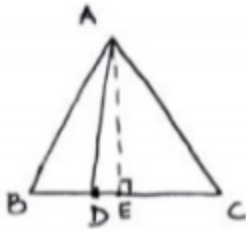
$\therefore$ Number are 2, 6, 10, 14 or 14, 10, 6, 2.

25. In an equilateral $\triangle ABC$, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Solution. Draw $AE \perp BC$



$\triangle AEB \cong \triangle AEC$ (RHS congruence rule)

$$\therefore BE = EC = \frac{1}{2} BC = \frac{1}{2} AB$$

Let $AB = BC = AC = x$

$$\text{Now } BE = \frac{x}{2} \text{ and } DE = BE - BD$$

$$= \frac{x}{2} - \frac{x}{3}$$

$$= \frac{x}{6}$$

$$\left. \begin{array}{l} \text{Now } AB^2 = AE^2 + BE^2 \dots (1) \\ \text{and } AD^2 = AE^2 + DE^2 \dots (2) \end{array} \right\}$$

$$\text{From (1) and (2) } AB^2 - AD^2 = BE^2 - DE^2$$

$$\Rightarrow x^2 - AD^2 = \left(\frac{x}{2}\right)^2 - \left(\frac{x}{6}\right)^2$$

$$\Rightarrow AD^2 = x^2 - \frac{x^2}{4} + \frac{x^2}{36}$$

$$\Rightarrow AD^2 = \frac{28}{36} x^2$$

$$\Rightarrow 9AD^2 = 7AB^2$$

OR

Given: A right triangle, $\triangle ABC$ right angled at B.

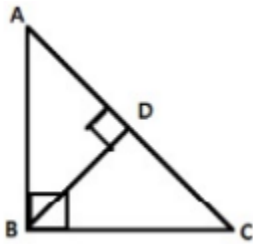
To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $AD \perp AC$.

We need to use the following theorem to prove the above result:

Theorem 1: If a perpendicular is drawn from the vertex of a right angle of a right triangle to the hypotenuse, then triangles on each side of the perpendicular are similar to each other and to the whole triangle.

The figure is shown below:



Using the theorem 1 stated above, we get

$$\triangle ABD \sim \triangle ACB$$

Since the corresponding sides of similar triangles are proportional, we have

$$\frac{AD}{AB} = \frac{AB}{AC}$$

Cross multiply to get,

$$AB^2 = AC \times AD \dots\dots(1)$$

Again, using the theorem 1 stated above, we get

$$\triangle BDC \sim \triangle ABC$$

Since the corresponding sides of similar triangles are proportion, we have

$$\frac{BC}{AC} = \frac{DC}{BC}$$

Cross multiply to get,

$$BC^2 = AC \times DC \dots (2)$$

Add (1) and (2) to get,

$$\begin{aligned} AB^2 + BC^2 &= AC \times AD + AC \times DC \\ &= AC \times (AD + DC) \\ &= AC \times AC \\ &= AC^2 \end{aligned}$$

Thus it is proved that,

$$AB^2 + BC^2 = AC^2$$

26. Draw a triangle ABC with BC = 6 cm, AB = 5 cm and $\angle ABC = 60^\circ$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the $\triangle ABC$.

Solution. A $\triangle A'BC'$ whose sides are $\frac{3}{4}$ of the corresponding sides $\triangle ABC$ can be drawn as follows.

step 1

draw a $\triangle ABC$ with side BC = 6cm, AB = 5cm and $\angle ABC = 60^\circ$.

step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

step 3

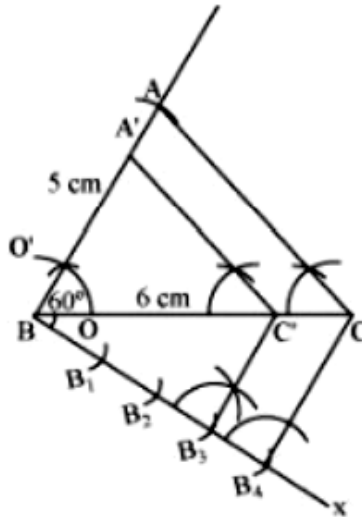
Locate 4 points (as 4 is greater in 3 and 4), B_1, B_2, B_3, B_4 , on line segment BX.

step 4

Join B_4C and draw a line through B_3 , parallel to B_4C intersecting BC at C' .

step 5

Draw a line through C' parallel to AC intersecting AB at A'. $\Delta A'BC'$ is the required triangle.



Justification

The construction can be justified by proving

$$A'B = \frac{3}{4}AB, BC' = \frac{3}{4}BC, A'C' = \frac{3}{4}AC$$

In $\triangle A'BC'$ and $\triangle ABC$

$$\angle A'BC' = \angle ACB \text{ (Corresponding angles)}$$

$$\angle A'BC' = \angle ABC \text{ (common)}$$

$$\therefore \Delta A'BC' \sim \Delta ABC \text{ (A similarity criterion)}$$

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} \dots\dots\dots(1)$$

In $\triangle BB_3C'$ and $\triangle BB_4C$,

$$\angle B_3BC' = \angle B_4BC \text{ (Common)}$$

$$\angle BB_3C' = \angle BB_4C \text{ (Corresponding angles)}$$

$$\Rightarrow \frac{BC'}{BC} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC'}{BC} = \frac{3}{4} \dots\dots\dots (2)$$

From equations (1) and (2), we obtain

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$$

$$\Rightarrow A'B = \frac{3}{4} AB, BC' = \frac{3}{4} BC, A'C' = \frac{3}{4} AC$$

This justifies the construction.

27. Prove that: $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

Solution. LHS $= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A(1 - 2(1 - \cos^2 A))}{\cos A(2\cos^2 A - 1)}$$

$$= \tan A \frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$$

$$= \tan A = RHS$$

28. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find :

i. The area of the metal sheet used to make the bucket.

ii Why we should avoid the bucket made by ordinary plastic ? [Use $\pi = 3.14$]

Solution. Here $r_1 = 15\text{cm}$, $r_2 = 5\text{cm}$ and $h = 24\text{ cm}$

(i) Area of metal sheet = CSA of the bucket + area of lower end

$$= \pi l (r_1 + r_2) + \pi r_2^2$$

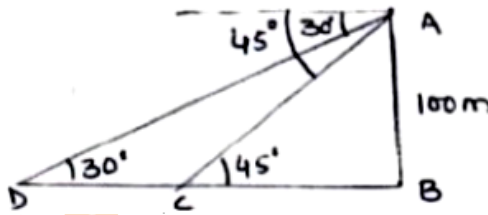
Where $l = \sqrt{24^2 + (15+5)^2} = 26\text{cm}$

$$\begin{aligned} \therefore \text{Surface area of metal sheet} &= 3.14(26 \times 20 + 25) \text{cm}^2 \\ &= 1711.3 \text{cm}^2 \end{aligned}$$

(ii) we should avoid use of plastic because it is non-degradable or similar value.

29. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

Solution. Let AB be the tower and ships are at points C and D.



$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{AB}{BC} = 1$$

$$\Rightarrow AB = BC$$

Also $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BC + cd}$

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$$\begin{aligned}\Rightarrow \frac{1}{\sqrt{3}} &= \frac{AB}{AB+CD} \\ \Rightarrow AB+CD &= \sqrt{3}AB \\ \Rightarrow CD &= AB(\sqrt{3}-1) \\ &= 100 \times (1.732-1) \\ &= 73.2m.\end{aligned}$$

30. The mean of the following distribution is 18. Find the frequency f of the class 19 – 21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	F	5	4

OR

The following distribution gives the daily income of 50 workers of a factory :

Daily Income (in Rs)	100-120	120-140	140-160	160-180	180-200
Numbers of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Solution.

Class	X	F	Fx
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	20f

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21-23	22	5	110
23-25	24	4	96
		40+f	704+20f

Mean

$$= 18 = \frac{704 + 20f}{40 + f}$$

$$\Rightarrow 720 + 18f = 704 + 20f$$

$$\Rightarrow f = 8$$

OR

Cumulative frequency distribution table of less than type is

Daily income	Cumulative frequency
Less than 100	0
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

