Solved Question Paper 2016
Class 10
Summative Assessment II
Subject - Mathematics

Time allowed: 3 hours **Maximum Marks: 90**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 31 questions divided into four sections - A, B, C and D.

(iii) Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.

(iv) Use of calculators is not permitted.

Section – A

Question1. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

Solution:

There are 26 red cards in a deck of 52 cards
Also there are 4 queens in a deck of 52 cards, 2 queens are black and 2 are red.
Now, numbers of card which are neither red nor queen are = 52 – (26 + 2) = 24.
$\therefore$ Probability of getting neither a red card nor a queen is: $=\frac{24}{52}=\frac{6}{13}$.

Question2. A ladder, leaning against a wall, makes an angle of 60^0 with the horizontal. If the foot of the ladder is 2.5m away from the wall, find the length of Ladder.

Solution:

Let AC be a ladder, AB is the wall and BC is the ground. The situation given in the question is shown in the figure given below

We know that
$\cos 60^0 = \frac{BC}{AC}$
$\Rightarrow \frac{1}{2} = \frac{2.5}{AC}$
$\Rightarrow AC = 5m$
Therefore, the length of ladder is 5 m.

Question3. In figure1, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30^0$, find $\angle PCA$.

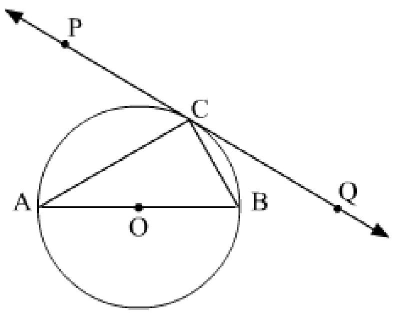
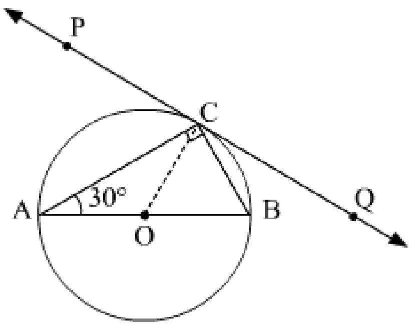


Figure 1

Solution:

Construction: Join OC



Now in Δ AOC
AO = OC (radius of the same circle)
So, $\angle OAC = \angle OCA = 30^0$
Also, $\angle OCP = 90^0$
Therefore, $\angle PCA = 90^0 - 30^0 = 60^0$.

Question4. For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P.?

Solution:

If three terms x, y and z are in A.P. then, $2y = x + z$
Since $k + 9$, $2k - 1$ and $2k + 7$ are in A.P.
$\therefore 2(2k-1) = (k+9) + (2k+7)$
$\Rightarrow 4k - 2 = 3k + 16$
$\Rightarrow k = 18.$

Section – B

Question5. In figure 2, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S. Prove that AB + CD = BC + DA.

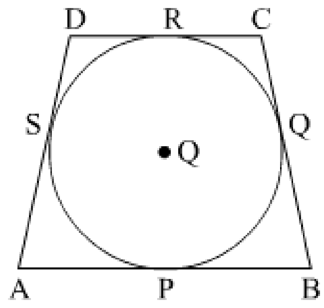


Figure 2

Solution:

Since tangents drawn from the exterior point to a circle are equal in length.

As, DR and DS are tangents from exterior point D so, DR = DS …… (1)

As, AP and AS are tangents from exterior point A so, AP = AS …….. (2)

As, BP and BQ are tangents from exterior point B so, BP = BQ ……... (3)

As, CR and CQ are tangents from exterior point C so, CR = CQ …….. (4)

Adding ...(1), ...(2), ...(3) and ...(4), we get

DR + AP + BP + CR = DS + AS + BQ + CQ

(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)

CD + AB = DA + BC

AB + CD = BC + DA

Hence proved.

Question6. Prove that the points (3, 0), (6, 4) and (–1, 3) are the vertices of a right angled isosceles triangle.

Solution:

Suppose, A (3, 0), B (6, 4) and C (–1, 3) be the vertices of the triangle.
From distance formula,
$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$
$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$
$CA = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$
As,
$5^2 + 5^2 = (5\sqrt{2})^2$
$\Rightarrow AB^2 + CA^2 = BC^2$
This implies that triangle ABC is a right angled isosceles triangle.

Question7. The 4^{th} term of an A.P is zero. Prove that the 25^{th} term of the A.P is three times its 11^{th} term.

Solution:

Let a be the first term and d be the common difference of an A.P. having n terms.
Its n^{th} term is given by, $a_n = a + (n-1)d$.
According to the question,
$a_4 = a + 3d = 0$
$\Rightarrow a + 3d = 0$
$\Rightarrow a = -3d$
$a_{25} = a + 24d$
$\Rightarrow a_{25} = -3d + 24d$
$\Rightarrow a_{25} == 21d$............(1)
$a_{11} = a + 10d$
$\Rightarrow a_{11} = -3d + 10d$
$\Rightarrow a_{11} = 7d$.............(2)
From equation (1) and (2)
$a_{25} = 3a_{11}$
Hence proved.

Question8. Let P and Q be the points of trisection of the line segment joining the points A (2, –2) and B (–7, 4) such that P is nearer to A. Find the coordinates of P and Q.

Solution:

Since P and Q be the points of trisection of the line segment joining the points A (2, –2) and B (–7, 4) such that P is nearer to A.
Therefore P divides line segment in the rail 1:2 and Q divides in 2:1

By section formula
Coordinates of P are
$$\left(\frac{1\times(-7)+2\times(2)}{1+2},\frac{1\times(4)+2\times(-2)}{1+2}\right)$$
$$=\left(\frac{-3}{3},\frac{0}{3}\right)$$
$$=(-1,0)$$
Coordinates of Q are
$$\left(\frac{2\times(-7)+1\times(2)}{2+1},\frac{2\times(4)+1\times(-2)}{2+1}\right)$$
$$=\left(\frac{-12}{3},\frac{6}{3}\right)$$
$$=(-4,2)$$

Question9. In figure 3, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = $2r$, show that $\angle$OTS = $\angle$OST = 30^0

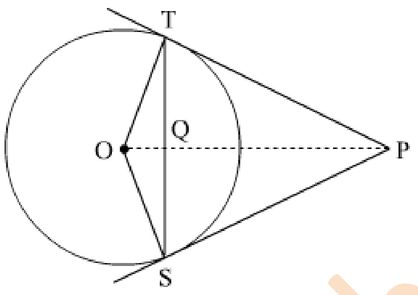


Figure 3

Solution:

In ΔOTP, $\angle OTP = 90^0$ (tangent to a circle perpendicular to the radius through the point of contact)
$$\sin\angle OPT=\frac{\text{Perpendicular}}{\text{Hypotenus}}=\frac{OT}{OP}$$
$$\Rightarrow \sin\angle OPT=\frac{r}{2r}$$
$$\Rightarrow \sin\angle OPT=\frac{1}{2}$$
$$\Rightarrow \angle OPT=30^0$$

Also,
$\angle$ OTP + $\angle$OPT + $\angle$TOP = 180^o (sum of angles of a triangle)
$\Rightarrow$ 90^o + 30^o + $\angle$TOP = 180^o
$\Rightarrow\angle$TOP = 180^o – 120^o = 60^o

In ΔOTP and ΔOSP
$OT = OS$ (radius)
$PT = PS$ (tangents from same external point)
$OP = OP$ (common)
So, $\Delta OTP \cong \Delta OSP$
Therefore,
$\angle$TPO = $\angle$SPO
$\Rightarrow$ OP is bisector of $\angle$TPS.
Now, ΔTQP and ΔSQP will also be congruent by SAS congruency.
Therefore, $\angle$OQT = OQS = 90^o.
In Δ OTQ,
$\angle$OQT + $\angle$OTQ + $\angle$TOQ = 180^o (angles of a triangle)
$\Rightarrow$ 90^o + $\angle$OTQ + 60^o = 180^o
$\Rightarrow$ $\angle$OTQ = 30^o $\Rightarrow$ $\angle$OTS = 30^o.
Similarly, $\angle$OST = 30^o.

Question10. Solve for x: $\sqrt{6x+7}-(2x-7)=0$

Solution:

We have
$$\sqrt{6x+7}-(2x-7)=0$$
$$\Rightarrow \sqrt{6x+7}=(2x-7)$$
Squaring both sides
$$6x+7=(2x-7)^2$$
$$\Rightarrow 6x+7=4x^2+49-28x$$
$$\Rightarrow 4x^2-34x+42=0$$
$$\Rightarrow 2x^2-17x+21=0$$
Using quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\Rightarrow x=\frac{17\pm\sqrt{17^2-(4\times2\times21)}}{2\times2}$$
$$\Rightarrow x=\frac{17\pm\sqrt{289-168}}{4}$$
$$\Rightarrow x=\frac{17\pm\sqrt{121}}{4}$$
$$\Rightarrow x=\frac{17\pm11}{4}$$
$$\Rightarrow x=\frac{28}{4},\frac{6}{4}$$
$$\Rightarrow x=7,1.5$$

Section – C

Question11. A Conical vessel with base of radius 5cm and height 24 cm, is full of water. This water is emptied into cylindrical vessel of base 10cm. Find the height to which water will rise in the cylindrical vessel.

Solution:

Let r, h, l be the radius, height and slant height of the cone.
Let R and H be the radius and height of the cylinder.
Since volume will remain same
Volume of water in conical vessel = Volume in cylindrical vessel
Therefore

$$\frac{1}{3}\pi r^2h=\pi R^2H$$
$$\frac{1}{3}\pi5^2(24)=\pi10^2H$$
$$H=2\text{ cm.}$$

Question12. In figure 4, O is the centre of a circle such that diameter AB = 13cm and AC = 12 cm. BC is joined. Find the area of shaded region.

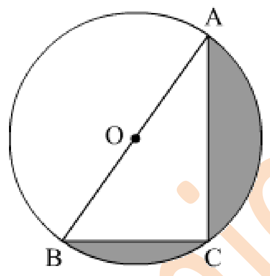


Figure 4

Solution:

In Δ ABC
$\angle C=90^0$ (angle in a semicircle)
therefore,

$$BC^2+AC^2=AB^2$$
$$\Rightarrow BC^2+12^2=13^2$$
$$\Rightarrow BC^2=169-144$$
$$\Rightarrow BC^2=25$$
$$\Rightarrow BC=5$$

$$\text{Area of shaded region}=\text{Area of semicircle}-\text{Area of triangle}$$
$$\Rightarrow \text{Area of shaded region}=\frac{1}{2}\pi(\text{radius})^2-\frac{1}{2}\times(base)\times(height)$$
$$\Rightarrow \text{Area of shaded region}=\left(\frac{1}{2}\times\frac{22}{7}\times\frac{13}{2}\times\frac{13}{2}\right)-\left(\frac{1}{2}\times5\times12\right)$$
$$\Rightarrow \text{Area of shaded region}=\frac{11}{7}\times\frac{13}{2}\times\frac{13}{2}-30$$
$$\Rightarrow \text{Area of shaded region}=36.39\,\text{cm}^2$$

Question13. If the point P(x, y) is equidistant from the points A ($a+b$, $b-a$) and B ($a-b$, $a+b$). Prove that $b\,x=a\,y$.

Solution:

Since the point P (x, y) is equidistant from the points A ($a+b$, $b-a$) and B ($a-b$, $a+b$)
$\therefore PA=PB$
From distance formula

$$\Rightarrow\sqrt{(x-(a+b))^2+(y-(b-a))^2}=\sqrt{(x-(a-b))^2+(y-(a+b))^2}$$
$$\Rightarrow(x-(a+b))^2+(y-(b-a))^2=(x-(a-b))^2+(y-(a+b))^2$$
$$\Rightarrow x^2+(a+b)^2-2x(a+b)+y^2+(b-a)^2-2y(b-a)=x^2+(a-b)^2-2x(a-b)+y^2+(a+b)^2-2y(b+a)$$
$$\Rightarrow-2x(a+b)-2y(b-a)=-2x(a-b)-2y(b+a)$$
$$\Rightarrow-2ax-2bx-2by+2ay=-2ax+2bx-2by-2ay$$
$$\Rightarrow-4bx=-4ay$$
$$\Rightarrow bx=ay$$

Question14. In figure 5, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8m, find the cost of canvas needed to make the tent if the canvas is available at the rate of 500/Sq meters.

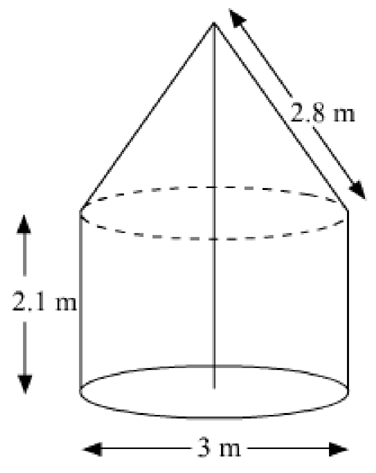


Figure 5

Solution:

Canvas needed to make the tent = Curved surface area of the conical part + curved surface area of cylindrical part.
Radius of the conical part = radius of cylindrical part = r =3/2 m
Slant height of the conical part is = l = 2.8 m.
Height of cylindrical part, h = 2.1m

Curved surface area of conical part = $\pi rl = \frac{22}{7}\times\frac{3}{2}\times 2.8$

Curved surface area of cylindrical part = $2\pi rh = 2\times\frac{22}{7}\times\frac{3}{2}\times 2.1$

Total surface area =

$$= \frac{22}{7}\times\frac{3}{2}\times 2.8m^2 + 2\times\frac{22}{7}\times\frac{3}{2}\times 2.1$$

$$= \left(\frac{22}{7}\times\frac{3}{2}\times 2.8 + 2\times\frac{22}{7}\times\frac{3}{2}\times 2.1\right)$$

$$= \frac{22}{7}\times\frac{3}{2}\times 7$$

$$= 33 \text{ m}^2$$

Therefore cost of canvas used =33 × 500 = Rs.16500.

Question15. A sphere of diameter 12 cm is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$.Find the diameter of the cylindrical vessel.

Solution:

Increase in the height of water level in the cylindrical vessel due to sphere (h) $=3\frac{5}{9}$ =32/9 cm.

Radius of sphere (R) = 6cm
Let radius of cylinder be r
Rise in volume of water in cylinder = Volume of sphere

$$\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow r^2\left(\frac{32}{9}\right) = \frac{4}{3}(6)^3$$

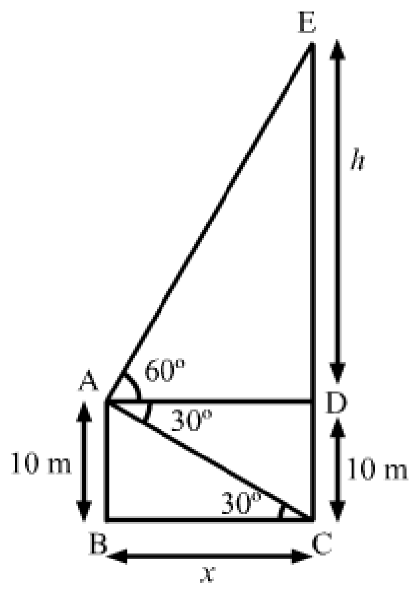
$$\Rightarrow r^2 = 81$$

$$\Rightarrow r = 9 \text{ cm}$$

Hence diameter of the cylindrical vessel is 2 × 9 =18 cm.

Question16. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60^0 and the angle of depression of the base of hill as 30^0. Find the distance of the hill from the ship and height of the hill.

Solution:



Suppose the man is standing on the deck of the ship at point A.
Let CE be the hill with base at C.
It is given that the angle of elevation of point E from A is 60^0 and the angle of depression of point C from A is 30^0
So,
$\angle$DAE=60^0
$\angle$CAD=30^0
$\angle$CAD=$\angle$ACB=30^0 (alternate angles)
AB =10 m
Suppose ED = h m and BC = x m

In ΔEAD, we have

$\tan 60^0 = \frac{ED}{AD}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$ (BC = AD = x)

$h = x\sqrt{3}$ ...(1)

In Δ ABC, we have

$\tan 30^0 = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$

$x = 10\sqrt{3}$ ...(2)

Therefore distance of the hill from the ship is $10\sqrt{3}\,m$

And height of the hill is

$h = x\sqrt{3} + 10$

$\Rightarrow h = \left(10\sqrt{3} \times \sqrt{3}\right) + 10$

$\Rightarrow h = 30\,\text{m}.$

Question17. In figure 6, find the area of the shaded region, enclosed between two concentric circles of radii 7cm and 14cm where ∠AOC=40^0

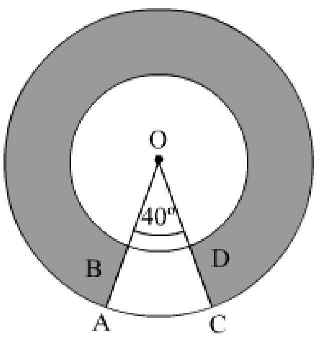
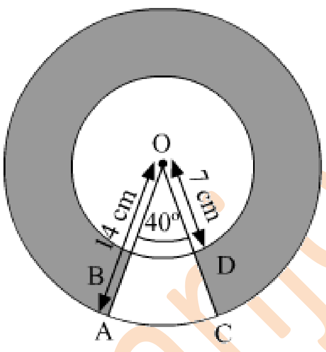


Figure 6

Solution:

We have



Area of shaded region = Area of circular ring – Area of region ABDC

$$= \left[\pi\left(14^2 - 7^2\right) - \frac{40}{360}\pi\left(14^2 - 7^2\right)\right]$$

$$= \pi\left(14^2 - 7^2\right)\left[1 - \frac{1}{9}\right]$$

$$= \frac{22}{7} \times (14+7) \times (14-7)\left[\frac{8}{9}\right]$$

$$= \frac{3696}{9}\,cm^2$$

$$= \frac{1232}{3}\,cm^2$$

$$= 410.67\ cm^2.$$

Question18. There are hundred cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card

(i) is divisible by 9 and is a perfect square.

(ii) is a prime number greater than 80.

Solution:

(i) Total cards =100

The numbers those are divisible by 9 and are perfect squares are 9, 36, 81.

Therefore probability that the number on the selected card is divisible by 9 and is a perfect square is $\frac{3}{100}$

(ii) Total cards =100

The prime numbers greater than 80 are,

83, 89, 97

Therefore probability that the number on the selected card is a prime number greater than 100 is $\frac{3}{100}$.

Question19. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of other two by 60.Find the numbers.

Solution:

Let x, x + 1 and x + 2 be the three consecutive numbers,

Then according to the question,

$(x+1)^2 = (x+2)^2 - x^2 + 60$
$\Rightarrow x^2 + 1 + 2x = x^2 + 4 + 4x - x^2 + 60$
$\Rightarrow x^2 - 2x - 63 = 0$
$\Rightarrow x^2 - (9-7)x - 63 = 0$
$\Rightarrow x^2 - 9x + 7x - 63 = 0$
$\Rightarrow x(x-9) + 7(x-9) = 0$
$\Rightarrow (x+7)(x-9) = 0$
$\Rightarrow x = -7, 9.$
The root –7 will be rejected, as natural number cannot be negative.
Therefore, the numbers are 9, 10, 11.

Question20. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P is 1 and their common differences are 1, 2 and 3. Prove that $S_1 + S_3 = 2S_2$.

Solution:

If a is the first term and d is the common difference of an A.P. having n terms then, the sum of its n terms is given by,

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

Therefore

$$S_1 = \frac{n}{2}\left(2\times 1 + (n-1)1\right)........(1)$$
$$\Rightarrow S_2 = \frac{n}{2}\left(2\times 1 + (n-1)2\right)$$
$$\Rightarrow S_2 = \frac{n}{2}\left(2 + 2n - 2\right)$$
$$\Rightarrow S_2 = n^2........(2)$$
$$S_3 = \frac{n}{2}\left(2\times 1 + (n-1)3\right).........(3)$$

Now from ....(1) and ...(3),

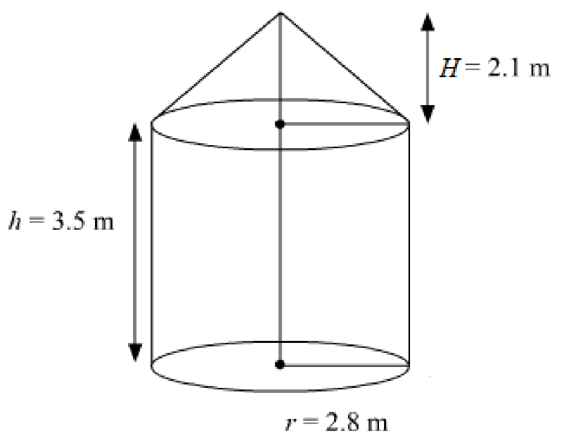
$$S_1 + S_3 = \left[\frac{n}{2}\left(2\times 1 + (n-1)1\right)\right] + \left[\frac{n}{2}\left(2\times 1 + (n-1)3\right)\right]$$
$$\Rightarrow S_1 + S_3 = \left[\frac{n}{2}\left(2 + n - 1\right)\right] + \left[\frac{n}{2}\left(2 + 3n - 3\right)\right]$$
$$\Rightarrow S_1 + S_3 = \left[\frac{n}{2}\left(n+1\right)\right] + \left[\frac{n}{2}\left(3n-1\right)\right]$$
$$\Rightarrow S_1 + S_3 = \frac{n}{2}\left(n + 1 + 3n - 1\right)$$
$$\Rightarrow S_1 + S_3 = \frac{n}{2}\left(4n\right)$$
$$\Rightarrow S_1 + S_3 = 2n^2$$
$$\Rightarrow S_1 + S_3 = 2S_2 \quad \text{from ....(3)}$$

Section – D

Question21. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8m and height 3.5m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs 120 per sq m, find the amount shared by each school to set up the tents. What value is generated by the above problem.

Solution:

The figure from the given information is as follows:



Let the radius of base of both cylinder and cone be r
Let the height of cone be H and that of cylinder be h.
Let the slant height of cone be l,
Then,
Area of tent = CSA of cylindrical base + CSA of cone

$= 2\pi rh + \pi rl$

$= 2\pi rh + \pi r\sqrt{H^2 + r^2}$

$= 2\pi rh + \pi r\sqrt{2.1^2 + 2.8^2}$

$= 2\pi rh + \pi r\sqrt{12.25}$

$= 2\pi rh + \pi r \times 3.5$

$= \frac{22}{7} \times 2.8(2 \times 3.5 + 3.5)$

$= \frac{22}{7} \times 2.8 \times 10.5$

$= 92.4 \text{ m}^2$

Cost of canvas used in making one tent = Rs. 92.4× 120 = Rs.11088
Cost of canvas used in making 1500 tents = Rs. 11088×1500= Rs.16632000
Share of one school = Rs.16632000/50 = Rs. 332640
The given problem shows kindness of school authorities.

Question22. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of *X* such that the sum of numbers of houses preceding the house numbered *X* is equal to sum of the number of houses following *X*.

Solution:

The number of houses preceding *X is X* – 1
Sum of the number of houses preceding the house numbered, *X* = 1 + 2 + 3+.... *X* – 1.
Here, *a* = 1, *d* =1
According to question

$S_{X-1} = S_{49} - S_X$

$\Rightarrow \frac{X-1}{2}\left(2a + (X-1-1)d\right) = \frac{49}{2}(2a + 48d) - \frac{X}{2}\left(2a + (X-1)d\right)$

$\Rightarrow \frac{X-1}{\cancel{2}}\left(2\times1 + (X-2)1\right) = \frac{49}{\cancel{2}}(2\times1 + 48\times1) - \frac{X}{\cancel{2}}\left(2\times1 + (X-1)1\right)$

$\Rightarrow (X-1)\left(2\times1 + (X-2)1\right) = 49(2\times1 + 48\times1) - X\left(2\times1 + (X-1)1\right)$

$\Rightarrow (X-1)X = 49\times50 - X(1+X)$

$\Rightarrow (X-1+1+X)X = 49\times50$

$\Rightarrow 2X^2 = 49\times50$

$\Rightarrow X^2 = 49\times25$

$\Rightarrow X = 7\times5$

$\Rightarrow X = 35$

Question23. In figure 7, the vertices of triangle are A (4, 6), B (1, 5) and C (7, 2). A line segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$.Calculate the area of Δ ADE and compare it with area of Δ ABC.

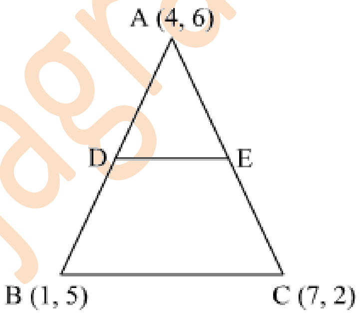
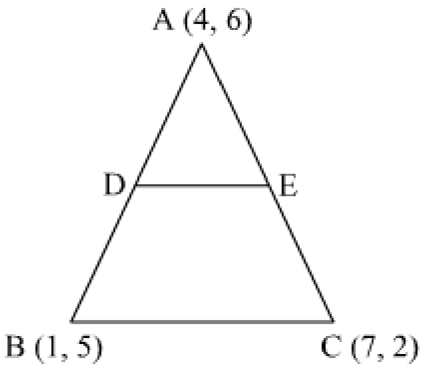


Figure: 7

Solution:

We have



Area of triangle ABC is:

$= \frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$

$= \frac{1}{2}\left|4(5-2) + 1(2-6) + 7(6-5)\right|$

$= \frac{15}{2} \text{sq.units}$

Now in Δ ADE and Δ ABC

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$ (given)

$\angle A = \angle A$ (common)

$\therefore \Delta ADE \sim \Delta ABC$

Therefore,

$$\frac{ar(ADE)}{ar(ABC)}=\left(\frac{AD}{AB}\right)^2=\left(\frac{1}{3}\right)^2$$

$$\Rightarrow ar(ADE)=\frac{ar(ABC)}{9}$$

$$\Rightarrow ar(ADE)=\frac{\left(15/2\right)}{9}$$

$$\Rightarrow ar(ADE)=\frac{5}{6}\text{ square units.}$$

Question24. In figure 8, two equal circles, with centre O and O' , touch each other at X.OO' produced meets the circle with centre O' at A.AC is tangent to the circle with centre O , at the point C.O'D is perpendicular to AC. Find the value of DO'/CO.

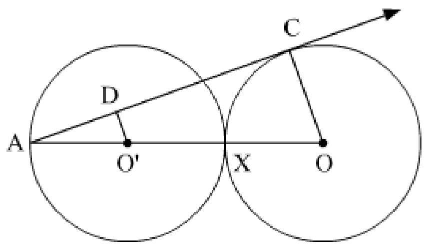


Figure 8

Solution:

As, circles are equal, so their radius are also equal.
That means $AO'=O'X=XO=r$
In $\Delta ADO'$ and ΔACO

$\angle ADO'=\angle ACO$

$\angle A=\angle A$ (common)

$\therefore \Delta ADO'\sim\Delta ACO$

Therefore

$$\frac{AO'}{AO}=\frac{DO'}{CO}$$

$$\Rightarrow \frac{r}{r+r+r}=\frac{DO'}{CO}$$

$$\Rightarrow \frac{r}{3r}=\frac{DO'}{CO}$$

$$\Rightarrow \frac{DO'}{CO}=\frac{1}{3}$$

Question25. A motor boat whose speed is 24 km/hr in still water takes 1 hr more to go 32km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution:

Let the speed of stream be x.
Then,
Speed of boat in upstream is $24 - x$
In downstream, speed of boat is $24 + x$
According to question,
Time taken in the upstream journey – Time taken in the downstream journey = 1 hour

$$\Rightarrow \frac{32}{24-x}-\frac{32}{24+x}=1$$

$$\Rightarrow \frac{24+x-24+x}{(24)^2-x^2}=\frac{1}{32}$$

$$\Rightarrow \frac{2x}{576-x^2}=\frac{1}{32}$$

$$\Rightarrow x^2+64x-576=0$$

$$\Rightarrow x^2+(72-8)x-576=0$$

$$\Rightarrow x^2+72x-8x-576=0$$

$$\Rightarrow x(x+72)-8(x+72)=0$$

$$\Rightarrow (x-8)(x+72)=0$$

$$\Rightarrow x=8,-72$$

Since speed cannot be negative,
So, speed of stream is 8km/hr.

Question26. In figure 9, is shown a sector OAP of a circle with centre O, containing $\angle\theta$ AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r[\tan\theta+\sec\theta+\frac{\theta\pi}{180}-1]$.

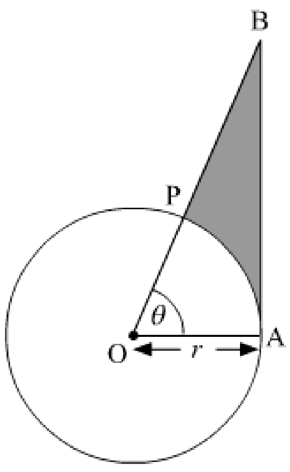


Figure 9

Solution:

In ΔAOB

$$\cos\theta = \frac{OA}{OB}$$

$$\Rightarrow OB = \frac{OA}{\cos\theta}$$

$$\Rightarrow OB = r\sec\theta$$

$$PB = OP - OB = r\sec\theta - r$$

Also,

$$\tan\theta = \frac{AB}{OA}$$

$$AB = OA\tan\theta$$

$$AB = r\tan\theta$$

$$\text{Length of arc AP} = \frac{\theta}{360}(2\pi r) = \frac{\pi\theta}{180}$$

$$\text{Perimeter of shaded region} = PB + AB + \text{Length of arc AP}$$

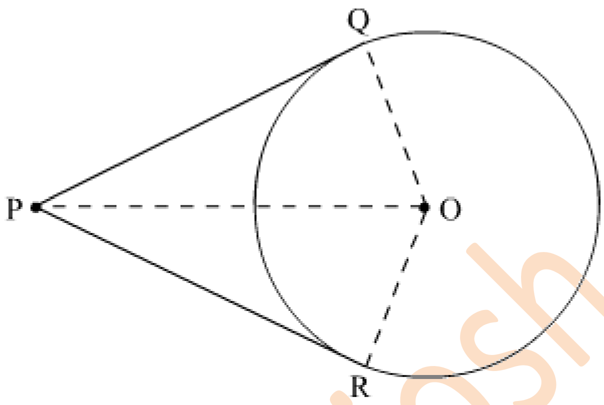
$$\text{Perimeter of shaded region} = \left(r\sec\theta - r + r\tan\theta + \frac{\pi\theta r}{180}\right)$$

$$\text{Perimeter of shaded region} = r\left(\sec\theta + \tan\theta + \frac{\theta\pi}{180} - 1\right)$$

Question27. Prove that the length of tangent drawn from an external point to a circle are equal.

Solution:

Let us draw a circle with centre O and two tangents PQ and PR are drawn from a external point P to the circle as shown in the figure given below,



Now

In ΔPQO and ΔPRO

$OQ = OR$ (radius)

$OP = OP$ (common)

$\angle OQP = \angle ORP$ (Each 90^0)

So,

$\Delta PQO \cong \Delta PRO$

therefore

$PQ = PR.$

Question28. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe fill the tank separately.

Solution:

Let one pipe takes x minutes to fill the tank,
Then tank filled in 1 min = $1/x$
Another pipe takes x + 5 minutes to fill the tank,
Then tank filled in 1 minutes = $1/(x+5)$

Time taken to fill the tank by running both pipes together = $11\frac{1}{9}$

Time taken to fill the tank in 1 min when both pipes are running together = $\frac{1}{11\frac{1}{9}}$

According to question

$$\frac{1}{x}+\frac{1}{x+5}=\frac{1}{11\frac{1}{9}}$$

$$\frac{x+5+x}{x^2+5x}=\frac{9}{100}$$

$$\frac{2x+5}{x^2+5x}=\frac{9}{100}$$

$$200x+500=9x^2+45x$$

$$\Rightarrow 9x^2-155x-500=0$$

$$\Rightarrow x=\frac{155\pm\sqrt{(155)^2-4\times9\times(-500)}}{2\times9}$$

$$\Rightarrow x=\frac{155\pm\sqrt{42025}}{18}$$

$$\Rightarrow x=\frac{155\pm205}{18}$$

$$\Rightarrow x=\frac{360}{18}\text{ or }-\frac{50}{18}$$

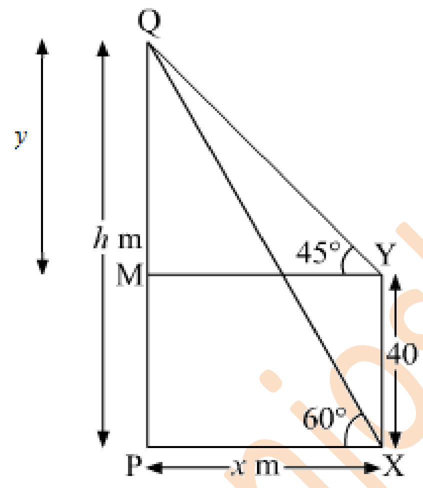
$$\Rightarrow x=20\text{ minutes}$$

Ignoring negative value as time cannot be negative.
One pipe takes fills tank in 20 minutes and other pipe will fill in 25 minutes.

Question29. From a point on the ground, the angle of elevation of the top of tower is observed to be 60^0, from a point 40 m vertically above the first point of observation, the angle of elevation of the top of the tower is 45^o. Find the height of the tower and its horizontal distance from the point of observation.

Solution:

Let PQ be the tower and X be the first point of observation on the ground, Y be the point 40 m vertically above X, the complete situation is shown in the figure given below,



Let QM = y, then in Δ QMY

$$\tan 45^0=\frac{y}{MY}$$

$$\Rightarrow 1=\frac{y}{MY}$$

$$\Rightarrow MY=y\text{.................(1)}$$

In ΔQPX

$$\tan 60^0=\frac{40+y}{PX}$$

$$\Rightarrow \sqrt{3}=\frac{40+y}{y}$$

$$\Rightarrow \sqrt{3}y=40+y$$

$$\Rightarrow \left(\sqrt{3}-1\right)y=40$$

$$\Rightarrow y=\frac{40}{\left(\sqrt{3}-1\right)}$$

$$\Rightarrow y=\frac{40}{\left(\sqrt{3}-1\right)}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$\Rightarrow y=\frac{40\left(\sqrt{3}+1\right)}{2}$$

$$\Rightarrow y=20\left(\sqrt{3}+1\right)m$$

Therefore,

Height of the tower is

$$h=40+20\left(\sqrt{3}+1\right)\text{m}$$

$$\Rightarrow h=\left(60+20\sqrt{3}\right)\text{m} = 94.64\text{ m}$$

Now,

$$\tan 60^0=\frac{h}{x}$$

$$\Rightarrow \sqrt{3}=\frac{94.64}{x}$$

$$\Rightarrow x=\frac{94.64}{\sqrt{3}}=54.64\text{ m.}$$

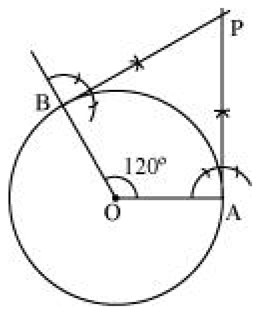
Horizontal distance = x = 54.64 m.

Question30. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60^0 to each other.

Solution:

Steps of construction:

1) Draw a circle of radius 4 cm with centre O.
2) Take a point A on the circumference of circle and join OA.
3) Draw a perpendicular to OA at A
4) Draw a radius OB, making an angle of 120^0 ($=180^0 - 60^0$) with OA
5) Draw a perpendicular to OB at B. Let these perpendiculars intersect at P.



Now PA and PB are required tangents.

Question31. A number x is selected at random from the numbers 1, 4, 9, 16 and another number y is selected at random from the numbers 1, 2, 3, 4. Find the probability of xy more than 16.

Solution:

Number x can be selected in four ways. Corresponding to each such way, there are four ways of selecting y.
Therefore two numbers can be selected in one of the following ways,
(1, 1), (1, 2), (1, 3), (1, 4)
(4, 1), (4, 2), (4, 3), (4, 4)
(9, 1), (9, 2), (9, 3), (9, 4)
(16, 1), (16, 2), (16, 3), (16, 4)
Therefore, the two numbers can be selected in 16 ways
The product xy can be more than 16 if x and y are chosen in following ways
(9, 2), (9, 3), (9, 4) ,(16, 2), (16, 3) (16, 4)
Therefore probability that the product xy is more than 16 is $= \frac{6}{16} = \frac{3}{8}$.