Solved Question Paper

2018

Class – XII

Subject – Mathematics

Delhi: Set – II

Time allowed: 3 hours **Maximum Marks: 100**

General Instructions:

i) All questions are compulsory.

ii) Please check that this Question Paper contains 26 Questions.

iii) Questions 1-4 in Section A are very short-answer type questions carrying 1 mark each.

iv) Questions 5-12 in Section B are long-answer I type questions carrying 2 marks each.

v) Questions 13-23 in Section C are long-answer I type questions carrying 4 marks each.

vi) Questions 24-29 in Section D are long-answer II type questions carrying 6 marks each.

SECTION A

Question 1:

If a*b denotes the larger ‘a’ and ‘b’ and if a o b = (a * b) + 3, then write the value of (5) o 10, where * and o are binary operations.

Solution 1:

Given: $a \circ b = (a * b) + 3$

Here $a * b$ denotes the larger of 'a' and 'b'.
Substitute $a = 5$ and $b = 10$ in $a \circ b = (a * b) + 3$
$5 \circ 10 = (5 * 10) + 3$

$= 10 + 3 \quad [5 * 10 = 10]$

$= 13$

Question 2:

Find the magnitude of each of the two vectors **a** and **b**, having the same magnitude such that the angle between them is 60^{o} and their scalar product is 9/2.

Solution 2:

Suppose $\vec{a}$ and $\vec{b}$ be the two vectors.

Now, $\vec{a}\,.\,\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

We are given that $|\vec{a}| = |\vec{b}|$, $\theta = 60^\circ$ and $\vec{a}\,.\,\vec{b} = \frac{9}{2}$

$$\therefore \frac{9}{2} = |\vec{a}||\vec{a}|\cos 60^\circ$$
$$\Rightarrow \frac{9}{2} = |\vec{a}|^2\left(\frac{1}{2}\right)$$
$$\Rightarrow |\vec{a}|^2 = 9$$
$$\Rightarrow |\vec{a}| = 3 = |\vec{b}|$$

Question 3:

If matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the value of ‘a’ and ‘b’

Solution 3:

Given: $A=\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

We know that if A is a skew symmetric matrix then, $A^T=-A$

$$\therefore \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$$

$$\therefore -a=2 \text{ and } -b=-3$$

$$\Rightarrow a=-2 \text{ and } b=3$$

Question 4:

Find the value of: $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

Solution 4:

Given: $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$

$$=\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$$

$$=\tan^{-1}\left(\tan\frac{\pi}{3}\right)-\cot^{-1}\left(\cot\frac{5\pi}{6}\right)$$

$$=\frac{\pi}{3}-\frac{5\pi}{6}$$

$$=\frac{2\pi-5\pi}{6}$$

$$=-\frac{\pi}{2}$$

SECTION B

Question 5:

The total cost C (x) associated with the production of x units of an item is given by C (x) = 0.005 x^3– 0.02 x^2 +30 x + 5000. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

Solution 5:

Given $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$

$$\begin{aligned}\text{Marginal Cost } &= \left\{\frac{d}{dx}\left[C(x)\right]\right\}_{x=3} \\ &= \left[0.015x^2 - 0.04x + 30\right]_{x=3} \\ &= 0.015(3)^2 - 0.04(3) + 30 \\ &= 30.015\end{aligned}$$

Question 6:

Differentiate $\tan^{-1}$ {(1 + cos x)/sinx}

Solution 6:

Given $\tan^{-1}\left(\dfrac{1+\cos x}{\sin x}\right)$

$$\begin{aligned}\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) &= \tan^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) \\ &= \tan^{-1}\left(\cot\frac{x}{2}\right) \\ &= \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right) \\ &= \frac{\pi}{2} - \frac{x}{2}\end{aligned}$$

Now,

$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)\right]=\frac{d}{dx}\left(\frac{\pi}{2}-\frac{x}{2}\right)=-\frac{1}{2}$$

Question 7:

Given $A=\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

Solution 7:

Given $A=\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$$\Rightarrow A^{-1}=\frac{1}{14-12}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}=\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Now,

$$\begin{aligned} \text{LHS} &= 2A^{-1} \\ &= 2\left(\frac{1}{2}\right)\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 9I - A \\ &= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 9-2 & 0-(-3) \\ 0-(-4) & 9-7 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \end{aligned}$$

Since, LHS = RHS

Hence proved.

Question 8: Prove that: $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right), x\in\left[-\frac{1}{2},\frac{1}{2}\right]$

Solution 8:

Given: $3\sin^{-1}x = \sin^{-1}\left(3x-4x^3\right)$

$$\begin{aligned}
&\text{LHS} = \sin^{-1}\left(3x-4x^3\right)\\
&\text{Substitute } x = \sin\theta\\
&\sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) = \sin^{-1}\left(\sin 3\theta\right)\\
&= 3\theta\\
&= 3\sin^{-1}x \qquad \left[\because x = \sin\theta \Rightarrow \theta = \sin^{-1}x\right]\\
&= \text{RHS}
\end{aligned}$$

Since, LHS = RHS

Hence proved.

Question 9: A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution 9:

Let A be the event when the sum of the observations is 8 = {(2, 6), (3, 5), (5, 3), (4, 4), (6, 2)}

$\therefore n(E) = 5$

Let B be the event when the observation on red die is less than 4 ={(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (5, 3), (6, 2)…….(6, 3)}

$\therefore n(B) = 18$

$\Rightarrow P(B) = \frac{18}{36} = \frac{1}{2}$

Now, $n(A\cap B) = 2$

$\Rightarrow n(A\cap B) = \frac{2}{36} = \frac{1}{18}$

$$\therefore P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{18}}{\frac{1}{2}}=\frac{1}{9}$$

Question 10: If θ is the angle between two vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$ find $\sin\theta$.

Solution 10:

Using cross product of two vectors

$$\left(\hat{i}-2\hat{j}+3\hat{k}\right).\left(3\hat{i}-2\hat{j}+\hat{k}\right)=\left|\hat{i}-2\hat{j}+3\hat{k}\right|\left|3\hat{i}-2\hat{j}+\hat{k}\right|\cos\theta$$

$$\Rightarrow \cos\theta=\frac{\left(\hat{i}-2\hat{j}+3\hat{k}\right).\left(3\hat{i}-2\hat{j}+\hat{k}\right)}{\left|\hat{i}-2\hat{j}+3\hat{k}\right|\left|3\hat{i}-2\hat{j}+\hat{k}\right|}$$

$$\Rightarrow \cos\theta=\frac{3+4+3}{\sqrt{1+4+9}\sqrt{9+4+1}}$$

$$\Rightarrow \cos\theta=\frac{10}{14}$$

$$\Rightarrow \cos\theta=\frac{5}{7}$$

$$\therefore \sin\theta=\sqrt{1-\cos^2\theta}$$

$$=\sqrt{1-\frac{25}{49}}$$

$$=\sqrt{\frac{24}{49}}$$

$$=\frac{2\sqrt{6}}{7}$$

Question 11: Find the differential equation representing the family of curves $y=ae^{bx+5}$ where a and b are arbitrary constants.

Solution 11:

Given: $y=ae^{bx+5}$

$$\Rightarrow \frac{y}{a} = e^{bx+5}$$

Differentiate $y = ae^{bx+5}$ with respect to x

$$\frac{dy}{dx} = ae^{bx+5}(b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{aby}{a} \qquad \left[\because \frac{y}{a} = e^{bx+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = by$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 \qquad \left[\because b = \frac{1}{y}\frac{dy}{dx}\right]$$

$$\Rightarrow y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

Question 12: Evaluate: $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

Solution 12:

Given: $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

$$\begin{aligned}\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x + 2\sin^2 x}{\cos^2 x} dx \\ &= \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x} dx \\ &= \int \frac{1}{\cos^2 x} dx \\ &= \int \sec^2 x dx \\ &= \tan x + C\end{aligned}$$

SECTION C

Question 13: If $y=\sin(\sin x)$ prove that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0.$

Solution 13:

Given: $y=\sin(\sin x)$

Differentiate it with respect to x

$$\frac{dy}{dx}=\cos(\sin x)\cos x$$

Again differentiate it with respect to x

$$\frac{d^2y}{dx^2}=-\sin(\sin x)\cos^2 x-\sin x\cos(\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2}+\sin(\sin x)\cos^2 x+\sin x\cos(\sin x)=0$$

$$\Rightarrow \frac{d^2y}{dx^2}+y\cos^2 x+\frac{\sin x}{\cos x}\frac{dy}{dx}=0 \qquad \begin{bmatrix} \because \frac{dy}{dx}=\cos(\sin x)\cos x \Rightarrow \cos(\sin x)=\frac{1}{\cos x}\frac{dy}{dx} \\ y=\sin(\sin x) \end{bmatrix}$$

$$\Rightarrow \frac{d^2y}{dx^2}+y\cos^2 x+\tan x\frac{dy}{dx}=0$$

$$\Rightarrow \frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0$$

Hence proved.

Question 14: Find the particular solution of the differential equation

$e^x\tan y\,dx+(2-e^x)\sec^2 y\,dy=0$, given that $y=\frac{\pi}{4}$ when $x=0$.

Solution 14:

Given: $e^x\tan y\,dx+(2-e^x)\sec^2 y\,dy=0$

$$\Rightarrow e^x \tan y dx = \left(e^x - 2\right)\sec^2 y dy$$

$$\Rightarrow \frac{e^x}{e^x - 2} dx = \frac{\sec^2 y}{\tan y} dy$$

Integrating both sides, we get

$$\int \frac{e^x}{e^x - 2} dx = \int \frac{\sec^2 y}{\tan y} dy$$

$$\Rightarrow \int \frac{dt}{t} = \int \frac{ds}{s} \quad \begin{bmatrix} \text{Let } e^x - 2 = t \text{ and } \tan y = s \\ \Rightarrow e^x dx = dt \text{ and } \sec^2 y dy = dx \end{bmatrix}$$

$$\Rightarrow \ln t = \ln s + \ln C$$

$$\Rightarrow t = sC$$

Substituting $t = e^x - 2$ and $s = \tan y$, we get

$$e^x - 2 = C \tan y$$

Now, $y = \frac{\pi}{4}$ when $x = 0$

$$e^0 - 2 = C\left(\tan\frac{\pi}{4}\right)$$

$$\Rightarrow -1 = C$$

$$\Rightarrow C = -1$$

$$\therefore e^x - 2 = -\tan y$$

$$\Rightarrow \tan y = 2 - e^x$$

$$\Rightarrow y = \tan^{-1}\left(2 - e^x\right)$$

Or

Question 14: Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$ given that $y = 0$ when $x = \frac{x}{3}$.

Solution 14:

Given: $\frac{dy}{dx}+2y\tan x=\sin x$

Comparing $\frac{dy}{dx}+2y\tan x=\sin x$ with general linear equation $\frac{dy}{dx}+Py=Q$, we get

$P=2\tan x$ and $Q=\sin x$

Now, integrating factor = $e^{2\int \tan x dx}=e^{2\log\sec x}=e^{\log\sec^2 x}=\sec^2 x$

The differential equation is given by

$$y\sec^2 x=\int \sin x\sec^2 x dx+C$$
$$\Rightarrow y\sec^2 x=\int \frac{\sin x}{\cos x}\sec x dx+C$$
$$\Rightarrow y\sec^2 x=\int \tan x\sec x dx+C$$
$$\Rightarrow y\sec^2 x=\sec x+C$$

Now, $y=0$ when $x=\frac{\pi}{3}$

$$(y)\sec^2\left(\frac{\pi}{3}\right)=\sec\left(\frac{\pi}{3}\right)+C$$
$$\Rightarrow C=-2$$

$$\therefore y\sec^2 x=\sec x-2$$

Integrating both sides, we get

$$\int\frac{e^x}{e^x-2}dx=\int\frac{\sec^2 y}{\tan y}dy$$
$$\Rightarrow \int\frac{dt}{t}=\int\frac{ds}{s}\quad\left[\begin{array}{l}\text{Let } e^x-2=t \text{ and } \tan y=s\\ \Rightarrow e^x dx=dt \text{ and } \sec^2 y dy=dx\end{array}\right]$$
$$\Rightarrow \ln t=\ln s+\ln C$$
$$\Rightarrow t=sC$$

Substituting $t=e^x-2$ and $s=\tan y$, we get

$$e^x-2=C\tan y$$

Now, $y=\frac{\pi}{4}$ when $x=0$

$$e^{0}-2=C\left(\tan\frac{\pi}{4}\right)$$
$$\Rightarrow -1=C$$
$$\Rightarrow C=-1$$

$$\therefore e^{x}-2=-\tan y$$
$$\Rightarrow \tan y=2-e^{x}$$
$$\Rightarrow y=\tan^{-1}\left(2-e^{x}\right)$$

Question 15: Find the shortest distance between the lines

$\vec{r}=\left(4\hat{i}-\hat{j}\right)+\lambda\left(\hat{i}-2\hat{j}-3\hat{k}\right)$ and $\vec{r}=\left(\hat{i}-\hat{j}-2\hat{k}\right)+\mu\left(2\hat{i}+4\hat{j}-5\hat{k}\right)$

Solution 15:

Here, $\vec{a}_1=4\hat{i}-\hat{j}, \vec{b}_1=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{a}_2=\hat{i}-\hat{j}+2\hat{k}, \vec{b}_2=2\hat{i}+4\hat{j}-5\hat{k}$

Now, $\vec{a}_2-\vec{a}_1=\hat{i}-\hat{j}+2\hat{k}-\left(4\hat{i}-\hat{j}\right)=-3\hat{i}+2\hat{k}$

Also,

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}$$
$$\Rightarrow \vec{b}_1\times\vec{b}_2=\left(-10+12\right)\hat{i}-\left(-5+6\right)\hat{j}+\left(4-4\right)\hat{k}$$
$$\Rightarrow \vec{b}_1\times\vec{b}_2=2\hat{i}-\hat{j}$$
$$\Rightarrow \left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{4+1}=\sqrt{5}$$

Therefore, the shortest distance is given by

$$d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|=\left|\frac{\left(2\hat{i}-\hat{j}\right).\left(-3\hat{i}+2\hat{k}\right)}{\sqrt{5}}\right|$$

$$=\left|\frac{-6}{\sqrt{5}}\right|$$

$$=\frac{6}{\sqrt{5}}\text{ units}$$

Question 16: Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X.

Solution 16:

The first five positive integers are 1, 2, 3, 4 and 5.

We can select two numbers from 5 numbers in ${}^5P_2=5\times4=20$

Now, let X denote the larger of two numbers, then X can take values from 2, 3, 4 and 5.

For X = 2, the possible observations are (1, 2) and (2, 1)

$$\therefore P\left(X=2\right)=\frac{2}{20}$$

For X = 3, the possible observations are (1, 3), (2, 3), (3, 1) and (3, 2)

$$\therefore P\left(X=3\right)=\frac{4}{20}$$

For X = 4, the possible observations are (1, 4), (2, 4), (3, 4), (4, 1), (4, 2) and (4, 3)

$$\therefore P\left(X=4\right)=\frac{6}{20}$$

For X = 5, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5) (5, 1), (5, 2), (5, 3) and (5, 4)

$$\therefore P\left(X=8\right)=\frac{8}{20}$$

Therefore, we have

X	2	3	4	5
P(X)	$\frac{2}{20}$	$\frac{4}{20}$	$\frac{6}{20}$	$\frac{8}{20}$

Therefore, Mean $\left[E(X)\right]=2\times\frac{2}{20}+3\times\frac{4}{20}+4\times\frac{6}{20}+5\times\frac{8}{20}$

$$=\frac{4}{20}+\frac{12}{20}+\frac{24}{20}+\frac{40}{20}$$
$$=\frac{80}{20}$$
$$=4$$

$$E\left(X^2\right)=2^2\times\frac{2}{20}+3^2\times\frac{4}{20}+4^2\times\frac{6}{20}+5^2\times\frac{8}{20}$$
$$=\frac{8}{20}+\frac{36}{20}+\frac{64}{20}+\frac{200}{20}$$
$$=\frac{340}{20}$$
$$=17$$

Variance = $E\left(X^2\right)-\left[E(X)\right]^2$

$$=17-(4)^2$$
$$=1$$

Therefore, mean and variance are 4 and 1 respectively.

Question 17: Using properties of determinates, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}=9\left(3xyz+zy+yz+zx\right)$$

Solution 17:

Given: $\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

Apply $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = \begin{vmatrix} -3y & 0 & 3x \\ 3y & -3z & 0 \\ 1 & 1+3z & 1 \end{vmatrix}$$

Taking 3 common from R_1 and R_2, we get

$$\begin{aligned}\begin{vmatrix} -3y & 0 & 3x \\ 3y & -3z & 0 \\ 1 & 1+3z & 1 \end{vmatrix} &= (3)(3)\begin{vmatrix} -y & 0 & x \\ y & -z & 0 \\ 1 & 1+3z & 1 \end{vmatrix} \\ &= 9\left[(-y)(-z) - 0 + x(y+3yz+z)\right] \\ &= 9\left[yz + xy + 3xyz + xz\right] \\ &= 9\left[3xyz + xy + yz + xz\right]\end{aligned}$$

Hence proved.

Question 18: Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1) where $x_1 = 2$ and $y_1 > 0$.

Solution 18:

Given: $16x^2 + 9y^2 = 145$

Substitute x = 2 in $16x^2 + 9y^2 = 145$

$$\begin{aligned}&16(2)^2 + 9y^2 = 145 \\ &\Rightarrow 64 + 9y^2 = 145 \\ &\Rightarrow 9y^2 = 81 \\ &\Rightarrow y = 3 \qquad (\text{Neglecting } y = -3 \text{ as } y > 0)\end{aligned}$$

Now, differentiate $16x^2+9y^2=145$ with respect to x which will give us the slope of the tangent.

$$32x+18y\frac{dy}{dx}=0$$
$$\Rightarrow\frac{dy}{dx}=-\frac{32x}{18y}$$

Substitute x = 2 and y = 3 in $\frac{dy}{dx}=-\frac{32x}{18y}$

$$\frac{dy}{dx}=-\frac{64}{54}=-\frac{32}{27}$$

The equation of tangent is given by

$$y-3=-\frac{32}{27}(x-2)$$
$$\Rightarrow 27y-81=-32x+64$$
$$\Rightarrow 32x+27y-145=0$$

The slope of the normal is $\frac{-1}{\text{Slope of tangent}}=\frac{27}{32}$

The equation of normal is given by

$$y-3=\frac{27}{32}(x-2)$$
$$\Rightarrow 32y-96=27x-54$$
$$\Rightarrow 27x-32y+42=0$$

OR

Question 18: Find the intervals in which the function $f(x)=\frac{x^4}{4}-x^3-5x^2+24x+12$ is (a) strictly increasing, (b) strictly decreasing.

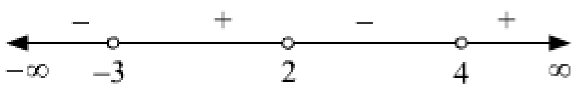
Solution 18:

Given: $f(x)=\frac{x^4}{4}-x^3-5x^2+24x+12$

$\Rightarrow f'(x) = x^3 - 3x^2 - 10x + 24$

Find the critical points by equation $f'(x)$ to zero.

$\therefore x^3 - 3x^2 - 10x + 24 = 0$
$\Rightarrow x^2(x-2) - x(x-2) - 12(x-2) = 0$
$\Rightarrow (x-2)(x^2 - x - 12) = 0$
$\Rightarrow (x-2)(x-4)(x+3) = 0$
$\Rightarrow x = -3,\ 2$ and 4
Therefore, we have the intervals $(-\infty, -3), (-3, 2), (2, 4)$ and $(4, \infty)$



(a) For $f(x)$ to be strictly increasing, we should have $f'(x) > 0$

$\Rightarrow (x-2)(x-4)(x+3) > 0$

Therefore, $x \in (-3, 2) \cup (4, \infty)$

(b) For $f(x)$ to be strictly decreasing, we should have $f'(x) < 0$

$\Rightarrow (x-2)(x-4)(x+3) < 0$

Therefore, $x \in (-\infty, -3) \cup (2, 4)$

Question 19: Find: $\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx$

Solution 19:

Given: $\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx$

Suppose $\sin x = t$
$\Rightarrow \cos x dx = dt$

$$\therefore \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx = \int \frac{2}{(1-t)(1+t^2)}dt$$

Using partial fraction, we get

$$\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$$

$$\Rightarrow 2 = A + At^2 + Bt + C - Bt^2 - Ct$$

$$\Rightarrow 2 = (A-B)t^2 + (B-C)t + A + C$$

$$\therefore A-B=0,\ B-C=0 \text{ and } A+C=2$$

Solving all these equations we get

$$A=B=C=1$$

Substituting the values, we get

$$2\int \frac{1}{(1-t)(1+t^2)}dt = \int \frac{1}{1-t}dt + \int \frac{t+1}{1+t^2}dt$$

$$= \int \frac{1}{1-t}dt + \int \frac{t}{1+t^2}dt + \int \frac{1}{1+t^2}dt$$

$$= \int \frac{1}{1-t}dt + \frac{1}{2}\int \frac{2t}{1+t^2}dt + \int \frac{1}{1+t^2}dt$$

$$= -\ln|1-t| + \frac{1}{2}\ln\left|1+t^2\right| + \tan^{-1} t + C$$

$$= -\ln|1-t| + \ln\left|\sqrt{1+t^2}\right| + \tan^{-1} t + C$$

$$= \frac{\ln\left|\sqrt{1+t^2}\right|}{\ln|1-t|} + \tan^{-1} t + C$$

Substitute back the value of t in $\dfrac{\ln\left|\sqrt{1+t^2}\right|}{\ln|1-t|} + \tan^{-1} t + C$.

$$\frac{\ln\left|\sqrt{1+\sin^2 x}\right|}{\ln|1-\sin x|} + \tan^{-1}(\sin x) + C$$

Question 20:

Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail' what is the probability that she threw 3, 4, 5 or 6 with the die?

Solution 20:

Let A_1 be the event when she get 1 or 2 and A_1 be the event when she get 3, 4, 5 or 6

$$\therefore P(A_1)=\frac{2}{6}=\frac{1}{3} \text{ and } P(A_2)=\frac{4}{6}=\frac{2}{3}$$

Let B be the event of getting exactly one tail.

$$\therefore P(B/A_1)=\frac{3}{8} \text{ and } P(B/A_2)=\frac{1}{2}$$

Required probability = $P(A_2/B)$

$$=\frac{P(A_2)P(B/A_2)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)}$$

$$=\frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}}$$

$$=\frac{\frac{1}{3}}{\frac{11}{24}}$$

$$=\frac{8}{11}$$

Question 21:

Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}, \vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}.\vec{a}=21$.

Solution 21:

We are given that $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$

$\vec{d}.\vec{b}=0$ and $\vec{d}.\vec{c}=0$

Let us suppose $\vec{d}=x\hat{i}+y\hat{j}+z\hat{k}$

Now,

$$\vec{d}.\vec{b}=0$$
$$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}-4\hat{j}+5\hat{k}\right)=0$$
$$\Rightarrow x-4y+5z=0 \qquad .....(1)$$
$$\vec{d}.\vec{c}=0$$
$$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$
$$\Rightarrow 3x+y-z=0 \qquad .....(2)$$
$$\vec{d}.\vec{a}=21$$
$$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(4\hat{i}+5\hat{j}-\hat{k}\right)=21$$
$$\Rightarrow 4x+5y-z=21 \qquad .....(3)$$

Solving (1), (2) and (3), we get

$$x=-\frac{1}{3}, y=\frac{16}{3} \text{ and } z=\frac{13}{3}$$

$$\therefore \vec{d}=-\frac{1}{3}\hat{i}+\frac{16}{3}\hat{j}+\frac{13}{3}\hat{k}$$

Question 22:

An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question.

Solution 22:

Let the dimension of the tank be *xxy*.

Therefore, the total surface area of the open tank would be $2\left(x^2+xy+xy\right)-x^2=x^2+4xy$

Let V be the volume of the tank.

$$V=x^2y$$
$$\Rightarrow y=\frac{V}{x^2}$$

Therefore, the surface area will be least when $x = 2y$

Question 23:

If $\left(x^2+y^2\right)^2=xy,$ find $\dfrac{dy}{dx}$

Solution 23:

Given: $\left(x^2+y^2\right)^2=xy$

$$\Rightarrow 2\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)=y+x\frac{dy}{dx}$$

$$\Rightarrow 4x\left(x^2+y^2\right)+4y\left(x^2+y^2\right)\frac{dy}{dx}=+x\frac{dy}{dx}$$

$$\Rightarrow 4x\left(x^2+y^2\right)-y=x\frac{dy}{dx}-4y\left(x^2+y^2\right)\frac{dy}{dx}$$

$$\Rightarrow 4x\left(x^2+y^2\right)-y=\left[x-4y\left(x^2+y^2\right)\right]\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4x\left(x^2+y^2\right)-y}{x-4y\left(x^2+y^2\right)}$$

Or

Question 23:

If $x=a\left(2\theta-\sin 2\theta\right)$ and $y=a\left(1-\cos 2\theta\right)$, find $\frac{dy}{dx}$ when $\theta=\frac{\pi}{3}$.

Solution 23:

Given: $x=a\left(2\theta-\sin 2\theta\right)$ and $y=a\left(1-\cos 2\theta\right)$

$$\Rightarrow \frac{dx}{d\theta}=a\left(2-2\cos 2\theta\right) \text{ and } \frac{dy}{d\theta}=2a\sin 2\theta$$

Now, $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$$=\frac{2a\sin 2\theta}{a\left(2-2\cos 2\theta\right)}$$

$$=\frac{\sin 2\theta}{1-\cos 2\theta}$$

$$\therefore \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{3}}=\frac{\sin 2\left(\frac{\pi}{3}\right)}{1-\cos 2\left(\frac{\pi}{3}\right)}=\frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}}=\frac{1}{\sqrt{3}}$$

SECTION D

Question 24: Evaluate : $\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x} dx$

Solution 24:

Given: $\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x} dx$

Let $\sin x - \cos x = t$

$\Rightarrow (\cos x + \sin x) dx = dt$

Squaring both sides in $\sin x - \cos x = t$

$(\sin x - \cos x)^2 = t^2$

$\Rightarrow \sin^2 x + \cos^2 x - \sin 2x = t^2$

$\Rightarrow \sin 2x = 1 - t^2$

$$\therefore \int \frac{\sin x + \cos x}{16 + 9\sin 2x} dx = \int \frac{dt}{16 + 9 - 9t^2}$$

$$= \int \frac{dt}{25 - 9t^2}$$

$$= \frac{1}{9} \int \frac{dt}{1 - \frac{9}{25} t^2}$$

$$= \frac{1}{9} \int \frac{dt}{1 - \left(\frac{3}{5} t\right)^2}$$

$$= \frac{1}{9} \left[\frac{3}{10} \log \left| \frac{\frac{3}{5} + t}{\frac{3}{5} - t} \right| \right]$$

Substitute $t = \sin x - \cos x$

$$\therefore \int \frac{\sin x + \cos x}{16 + 9\sin 2x} dx = \frac{1}{9} \left[\frac{3}{10} \log \left| \frac{\frac{3}{5} + \sin x - \cos x}{\frac{3}{5} - \sin x - \cos x} \right| \right]$$

$$\text{Now, } \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x} dx = \frac{1}{9} \left[\frac{3}{10} \log \left| \frac{\frac{3}{5} + \sin x - \cos x}{\frac{3}{5} - \sin x - \cos x} \right| \right]_0^{\pi/4}$$

$$= \frac{1}{30} \log 4$$

OR

Question 24: $\int_1^3 \left(x^2 + 3x + e^x\right) dx$, as the limit of the sum.

Solution 24:

Given: $\int_1^3 \left(x^2 + 3x + e^x\right) dx$

Here, $a = 1, b = 3\ f(x) = x^2 + 3x + e^x$

$$\therefore h = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$\int_1^3 \left(x^2 + 3x + e^x\right)dx = \lim_{h\to 0}\left[f(1) + f(1+h) + f(1+2h) + f(1+3h) \ldots\ldots + f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0}\left[\begin{matrix} 4 + e + (1+h)^2 + 3(1+h) + e^{1+h} + (1+2h)^2 + 3(1+2h) + e^{1+2h} + (1+3h)^2 + 3(1+3h) + e^{1+3h} + \ldots\ldots \\ \ldots + (1+(n-1)h)^2 + 3(1+(n-1)h) + e^{1+(n-1)h} \end{matrix}\right]$$

$$= \lim_{h\to 0}\left[4n + e + h^2\frac{(n-1)(n-2)(2n-3)}{6} + 2h\left(\frac{n^2-n}{2}\right) + 3h\left(\frac{n^2-n}{2}\right) + e\frac{e^h\left[e^{h(n-1)} - 1\right]}{e^h - 1}\right]$$

$$= \lim_{h\to 0}\left[4n + e + h^2\frac{(n-1)(n-2)(2n-3)}{6} + 2h\left(\frac{n^2-n}{2}\right) + 3h\left(\frac{n^2-n}{2}\right) + e\frac{e^h\left[e^{h(n-1)} - 1\right]}{e^h - 1}\right]$$

$$= \frac{62}{3} + e^3 - e$$

Question 25:

A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of Rs 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Formulate the above LPP and solve it graphically and find the maximum profit.

Solution 25:

Let the factory manufacture x screws of type A and y screws of type B on each day. Therefore,

$x \geq 0$ and $y \geq 0$

We can write the two equations as given below:

$4x + 6y \le 4 \times 60$

$\Rightarrow 4x + 6y \le 240$

and

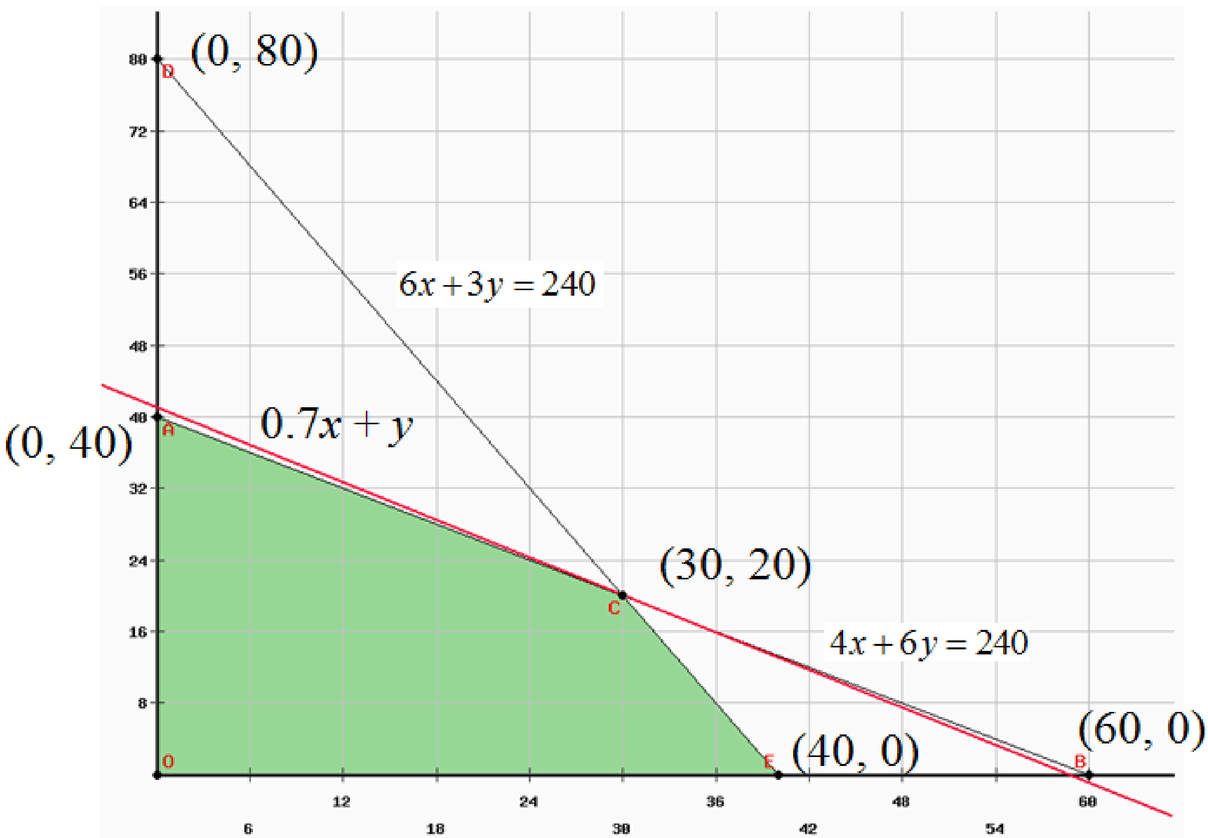
$6x + 3y \le 4 \times 60$

$\Rightarrow 6x + 3y \le 240$

Now, the total profit for A and B are 70 paise and 1 Rs.

Therefore, we have to maximize $0.7x + y$

First we draw the graph of $x \ge 0$, $y \ge 0$, $4x + 6y \le 240$ and $6x + 3y \le 240$



The value of $0.7x + y$ at the corner points

	x	y	Objective Function = $0.7x + y$
O	0	0	0
A	0	40	40
B	60	0	42
C	30	20	41
D	0	80	80
E	40	0	28

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 41.

Question 26:

Let $A=\{x\in Z:0\le x\le 12\}$. Show that $R=\{(a,b):a,b\in A,|a-b|$ is divisible by 4) is an equivalence relation. Find the set of all elements relates to 1. Also write the equivalence class [2].

Solution 26:

Given: $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Also, $R=\{(a,b):a,b\in A,|a-b|$

For reflexive

Substitute $a = b$

$R=|b-b|=0$ which is divisible by 4

Therefore, R is reflexive.

For symmetric

Substitute $a=b$ and $b=a$

$R=|b-a|=|a-b|$ which is divisible by 4

Therefore, R is symmetric.

For Transitive

If $|a-b|$ is divisible by 4 i.e., $a-b=\pm 4k$

Also, If $|b-c|$ is divisible by 4 i.e., $b-c=\pm 4k$

$\therefore |a-c|=\pm 4k\pm 4k$ is also divisible by 4

Therefore, R is Transitive.

Now, set of elements related to 1 is {(1, 1), (1, 5), (1, 9), (5, 1), (9, 1)}

Let $(x, 2)\in R$

$|x-2|=4k$ where $k\leq 3$

$\therefore x=2, 6, 10$

Therefore, the equivalence class [2] is {2, 6, 10}.

OR

Question 26:

Show that the function $f:\mathbb{R}\rightarrow\mathbb{R}$ defined by $f(x)=\dfrac{x}{x^2+1}, \forall x\in\mathbb{R}$ is neither one – one nor onto.

Also, if $g:\mathbb{R}\rightarrow\mathbb{R}$ is defined as $g(x)=2x-1$, find fog (x).

Solution 26:

Given: $f(x)=\dfrac{x}{x^2+1}$

Let $y=\dfrac{x}{x^2+1}$

$\Rightarrow yx^2+y=x$

$\Rightarrow yx^2+y-x=0$

$\Rightarrow x=-\dfrac{1\pm\sqrt{1-4y^2}}{2y}$

Since for any value of x we will get two values for y.

Hence, $f(x)$ is many-one not one-one.

Since x is real number.

Hence, $1-4y^2 \geq 0$

$$\Rightarrow (1+2y)(1-2y) \geq 0$$

$$\therefore \frac{-1}{2} \leq y \leq \frac{1}{2}$$

Therefore, we always get the value of y as $\left[\frac{-1}{2}, \frac{1}{2}\right]$

Hence, $f(x)$ is not onto.

Now, $g(x) = 2x-1$

$$fog(x) = \frac{2x-1}{(2x-1)^2+1}$$

$$= \frac{2x-1}{4x^2+1-4x+1}$$

$$= \frac{2x-1}{4x^2-4x+2}$$

Question 27:

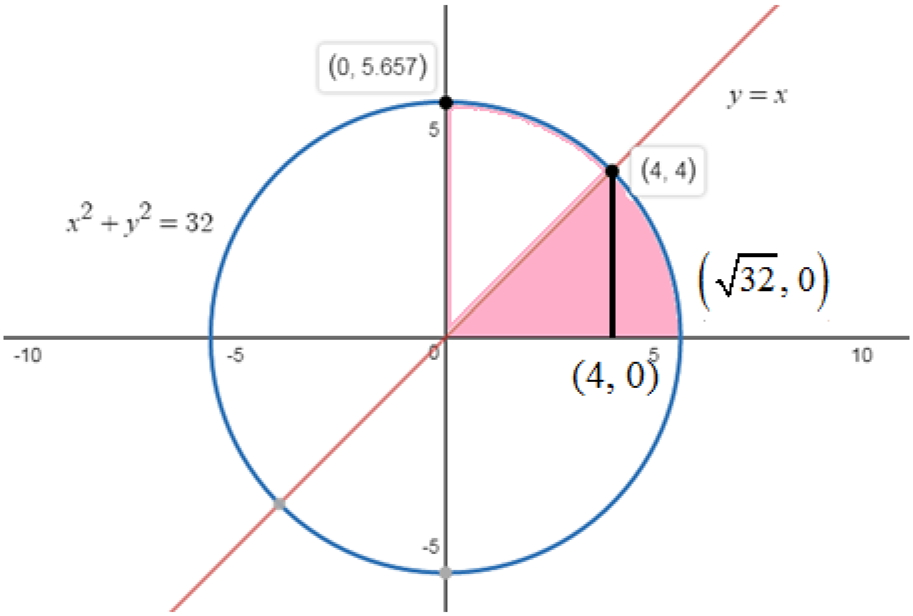
Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Solution 27:

Given: $y = x$ and $x^2 + y^2 = 32$

Substitute $y = x$ in $x^2 + y^2 = 32$, we get

$x^2 + x^2 = 32$
$\Rightarrow 2x^2 = 32$
$\Rightarrow x^2 = 16$
$\Rightarrow x = \pm 4$



Therefore, the required area is given by $\int_0^4 y dx + \int_4^{\sqrt{32}} |y| dx$

$= \int_0^4 x dx + \int_4^{\sqrt{32}} \sqrt{32 - x^2} dx$

$= \frac{1}{2}\left[x^2\right]_0^4 + \left[\frac{1}{2}x\sqrt{32 - x^2} + 16\sin^{-1}\left(\frac{x}{32}\right)\right]_4^{\sqrt{32}}$

$= 4\pi$ sq units

Question 28:

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the system of equations

$$2x-3y+5z=11$$
$$3x+2y-4z=-5$$
$$x+y-2z=-3$$

Solution 28:

Given: $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$\Rightarrow |A| = 2(-4+4)+3(-6+4)+5(3-2)$$
$$=-6+5$$
$$=-1$$

Now,

$$adj.A=\begin{bmatrix} (-1)^{1+1}\begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} \\ (-1)^{2+1}\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} & (-1)^{2+2}\begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} & (-1)^{2+3}\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} \\ (-1)^{3+1}\begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} & (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} & (-1)^{3+3}\begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} \end{bmatrix}^T$$

$$=\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T$$

$$=\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = -1\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

We can write the given equation as AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1,\ y = 2$ and $z = 3$

OR

Question 28:

Using elementary row transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}.$$

Solution 28:

Given: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

We know that $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = A\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Apply $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = A\begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Apply $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

Question 29:

Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

Solution 29:

Given: $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$

$\Rightarrow \vec{r} = (2 + 3\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$

Substitute $\vec{r} = (2 + 3\lambda)\hat{i} - (1 - 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$ in $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

$$\therefore \left[(2+3\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(2+2\lambda)\hat{k}\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$

$$\Rightarrow (2+3\lambda)-(-1+4\lambda)\hat{j}+(2+2\lambda)=5$$

$$\Rightarrow 2+3\lambda+1-4\lambda+2+2\lambda=5$$

$$\Rightarrow \lambda+5=5$$

$$\Rightarrow \lambda=0$$

Substituting $\lambda=0$ in $(2+3\lambda)\hat{i}+(-1-4\lambda)\hat{j}+(2+2\lambda)\hat{k}$ we get

$$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$$

Therefore, the coordinates of the points are $(2,-1,2)$ and $(-1,-5,-10)$.

The distance between the two points is given by

$$\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}=\sqrt{9+16+144}$$

$$=\sqrt{169}$$

$$=13 \text{ units}$$