Solved Practice Paper
2017
Class – XII
Subject – Physics
Set – I

Time allowed: 3 hours **Maximum Marks: 70**

General Instructions:

(a) All questions are compulsory.

(b) There are 26 questions in all. Question number 1 to 5 carry one mark each, question number 6 to 10 carry two marks each, question number 11 to 22 carry three marks each, question number 23 carry four marks and question number 24 to 26 carry 5 marks each.

(c) There is no overall choice. However, internal choices have been provided in some questions.

(d) You may use the following values of physical constants wherever necessary:

$$c = 3 \times 10^{8} \text{m/s}$$
$$h = 6.63 \times 10^{-34} \text{Js}$$
$$e = 1.6 \times 10^{-19} \text{C}$$
$$\mu_0 = 4\pi \times 10^{-7}\ \text{T m A}^{-1}$$
$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$$
$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ \text{N m}^2\text{C}^{-2}$$
$$m_e = 9.1 \times 10^{-31}\ \text{kg}$$
$$\text{Mass of neutron} = 1.675 \times 10^{-27}\ \text{kg}$$
$$\text{Mass of proton} = 1.673 \times 10^{-27}\ \text{kg}$$
$$\text{Avogadro's number} = 6.023 \times 10^{23}\ \text{per gram mole}$$
$$\text{Boltzmann constant} = 1.38 \times 10^{-23}\ \text{JK}^{-1}$$

Question1: What is the relation between electric current (I) and drift velocity (v_d) of electrons?

Sol:

Let n be the number density of free electrons in a conductor of length l and area of cross-section 'A', the drift velocity is given by, $I = n\, e\, A\, v_d$ where, e is the magnitude of charge on electrons.

Question2: Prove that $\frac{\text{volt}}{\text{metre}}$ and $\frac{\text{newton}}{\text{coulomb}}$ are the units of the same physical quantity.

Sol:

$$\because \text{Volt} = \frac{\text{joule}}{\text{coulomb}} \text{ and joule} = \text{newton} \times \text{metre}$$

$$\therefore \frac{\text{volt}}{\text{metre}} = \frac{\text{joule}/\text{coulomb}}{\text{metre}} = \frac{\text{newton} \times \text{metre}}{\text{coulomb} \times \text{metre}} = \frac{\text{newton}}{\text{coulomb}}.$$

Question3: Give two examples of Paramagnetic and Ferromagnetic material?

Sol:

Ferromagnetic material: Nickel, Cobalt

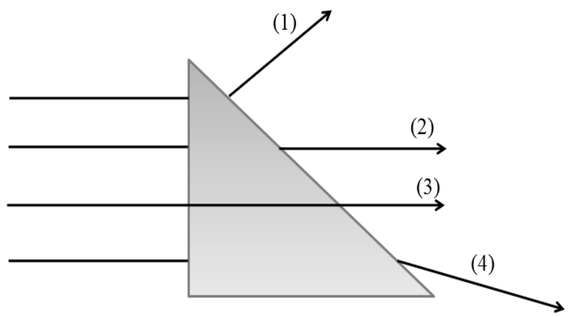
Paramagnetic material: Platinum, Aluminum

Question4: Name the network formed by joining 4 to 6 computers in an office?

Sol:

Local Area Network (LAN).

Question5: Figure given below shows four electric field lines passing through a metallic triangular lamina. Which one of the following four lines is shown correctly?



Sol:

The electric field line inside a conductor is zero. Also it is always perpendicular to the surface of the conductor. Therefore, line (1) is the correct representation.

Question6: How focal length of the lens will change if the red light incident on it replaced by violet light? Give reasons for your answer.

Sol:

As, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Where, μ is refractive index of the material

Now, $f \propto \frac{1}{(\mu - 1)}$ and $\mu_v > \mu_R$

The increase in refractive index would result in decrease of focal length of lens. So, by replacing red light with violet light, decreases the focal length of the lens used.

Question7: In a CE transistor amplifier, there is a current and voltage gain associated with the circuit. In other words there is a power gain. Considering power a measure of energy, does the circuit violate conservation of energy?

Sol:

In case of CE transistor amplifier, the power gain is very high. In this type of circuit, the extra power required for amplified output is obtained from DC source. Thus, the circuit used does not violet the law of conservation.

Question 8: The refractive index of a medium with respect to air is √3. What is the angle of refraction, if the unpolarized light is incident on it at the polarizing angle of the medium?

Sol:

Given, refractive index of the medium = $\mu = \sqrt{3}$.

Let angle of incident be i_p and angle of refraction be r.

We know that, $\mu = \tan i_p$ or $i_p = \tan^{-1}\mu$,

Therefore, $i_p = \tan^{-1}(\sqrt{3})$ or $i_p = 60^o$.

Therefore, $r = 90^o – 60^o = 30^o$.

OR

Question 8: What do you understand by resolving power of an optical instrument? Suppose a light of wavelength 3000 angstrom is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 254 cm?

Sol:

Resolving power of an optical instrument is the power or ability of the instrument to produce distinctly separate images of two closely objects.

Given, diameter of objective of telescope, D = 254 cm

Wavelength of light used = λ = 3000 angstrom = 3×10^{-5} cm.

Limit of resolution of a telescope = $d\theta = \frac{1}{d} = \frac{1.22\lambda}{d} = \frac{1.22 \times 3 \times 10^{-5}}{254} = 1.44 \times 10^{-7}$ radian.

Question9: What is attenuation? Name three different types of modulations used for a message signal using a sinusoidal continuous carrier wave.

Sol:

It is the loss of strength of a signal while propagating through a medium.

A sinusoidal carrier wave can be modulated in three ways:

(*i*) Amplitude modulation

(*ii*) Frequency modulation

(*iii*) Phase modulation.

Question 10: Show that Bohr's second postulate "The electron revolves around the nucleus only in certain fixed orbits without radiating energy" can be explained on the basis of de-Broglie hypothesis of wave nature of electron.

Sol:

Suppose an electron of mass *m* is confined to move on a path of length *l* with velocity *v*.

The de-Broglie wavelength associated with this electron is given by $\lambda = \frac{h}{mv}$ or $\lambda = \frac{h}{p}$

$$\Rightarrow p = \frac{h}{\lambda} = \frac{h}{\left(2l/n\right)} \text{ or } p = \frac{nh}{2l}.$$

If this electron revolved in a circular path of radius *r* then, $2l = 2\pi r$ or $l = \pi r$.

$$\Rightarrow p = \frac{nh}{2\pi r} \Rightarrow p \times r = n\left(\frac{h}{2\pi}\right)$$ or (angular momentum) = integral multiple of (h/2π).

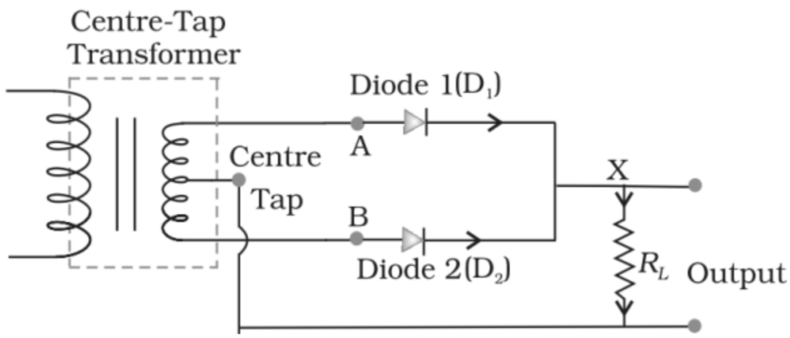
Question11: What is rectifier? Draw a circuit diagram of a full-wave rectifier. Explain its working principle.

Sol:

Rectifier is a device which is used for converting alternating current/voltage into direct current/voltage.

Principle: Its working is based on the fact that the resistanceof *p*-*n* junction becomes negligible when forward biased and becomes very high when reversed biased.

Circuit diagram of full wave rectifier:

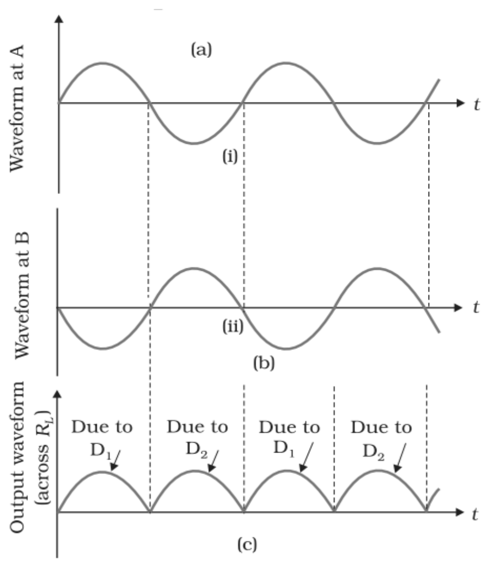


The circuit consists of two diodes as shown in the figure given above. It gives output rectified voltage corresponding to both the positive as well as negative half of the AC cycle. Hence, it is known as full-wave rectifier.

Here the *p*-side of the two diodes are connected to the ends of the secondary of the transformer. The *n*-side of the diodes are connected together and the output is taken between this common point of diodes and the midpoint of the secondary of the transformer.

So for a full-wave rectifier the secondary of the transformer is provided with a centre tapping and so it is called centre-tap transformer.

The voltage rectified by each diode is only half the total secondary voltage. Each diode rectifies only for half the cycle, but the two do so for alternate cycles. Thus, the output between their common terminals and the centretap of the transformer becomes a full-wave rectifier output.



Suppose the input voltage to A with respect to the centre tap at any instant is positive, at that instant, voltage at B being out of phase will be negative. So, diode D_1 gets forward biased and conducts (while D_2 being reversed biased is not conducting). Hence, during this positive half cycle we get an output current (and a output voltage across the load resistor R_L).

Question12: Two polaroids are oriented in such way that the intensity of the light transmitted by them is maximum. To what percentage of its maximum value is the intensity of transmitted light reduced when one of the polaroids is rotated through (*i*) 30^o (*ii*) 60^o.

Sol:

The two polaroids are parallel to each other initially. If I_1 is the intensity of light from the first polaroid, then the intensity of transmitted light by the second is: $I = I_o \cos^2 \theta = I_o \cos 0^o = I_o$.

(*i*) When $\theta = 30^o$ then, the transmitted intensity is $I = I_o \cos^2 30^o = I_o (\sqrt{3} / 2) = 0.75\, I_o$.

Which means transmitted intensity is reduced to 75 % of the maximum intensity.

(*ii*) When $\theta = 60^o$ then, the transmitted intensity is $I = I_o \cos^2 60^o = I_o (1 / 2)^2 = 0.25\, I_o$.

Which means transmitted intensity is reduced to 25 % of the maximum intensity.

Question 13: Write two basic modes of communication. Explain the process of amplitude modulation. Draw a schematic sketch showing how amplitude modulated signal is obtained by superposing a modulating signal over a sinusoidal carrier wave.

Sol:

The two basic modes of communication are:

a) Point-to-point communication

b) Broadcast communication

Amplitude Modulation: Amplitude modulation is produced by varying the amplitude of the carrier waves in accordance with the amplitude of the modulating wave.

Let the carrier wave be $c(t) = A_c \sin \omega_c t$ and the modulating signal be $m(t) = A_m \sin \omega_m t$, where $\omega_m = 2\pi f_m$ is the angular frequency of the message signal.

Modulated signal $c_m(t)$ is

$$c_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$$= A_c \left(1 + \frac{A_m}{A_c} \sin \omega_m t\right) \sin \omega_c t$$

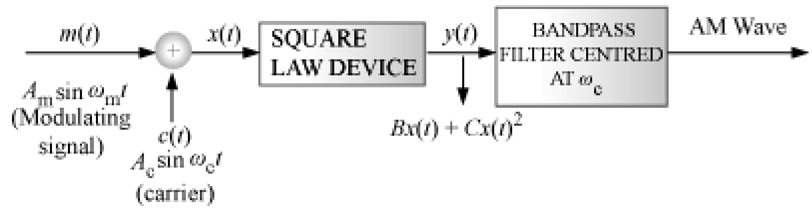
$$c_m(t) = A_c \sin \omega_c t + \mu A_m \sin \omega_m t \sin \omega_c t$$

Where $\mu = A_m / A_c$ is the modulation index

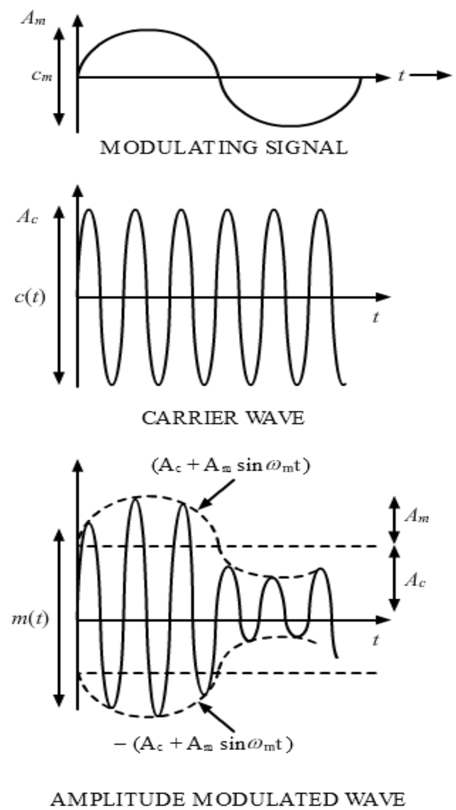
$$c_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m) t - \frac{\mu A_c}{2} \cos(\omega_c + \omega_m) t$$

$\omega_c - \omega_m$ and $\omega_c + \omega_m$ are the lower side and upper side frequencies, respectively.

Production of Amplitude Modulated Wave



Amplitude modulated signal obtained by superposing a modulating signal over a sinusoidal carrier wave, is shown in the figure given below:



Question 14: Define the terms 'threshold frequency' and 'stopping potential' in the study of photoelectric emission. Explain briefly the reasons why wave theory of light is not able to explain the observed features in photoelectric effect.

Sol:

Threshold Frequency: The minimum frequency of incident light which is just capable of ejecting electrons from a metal. It is denoted by v_o.

Stopping Potential: The minimum retarding potential applied to anode of a photoelectric tube which is just capable of stopping photoelectric current.

The observed characteristics of photoelectric effect could not be explained on the basis of wave theory of light due to the following reasons.

(***i***) According to wave theory, the light propagates in the form of wavefronts and the energy is distributed uniformly over the wavefronts. With increase of intensity of light, the amplitude of waves and the energy stored by waves will also increase. These waves will then, provide more energy to electrons of metal, consequently the energy of electrons will also increase.

Therefore, according to wave theory, the kinetic energy of photoelectrons must depend on the intensity of incident light; but according to experimental observations, the kinetic energy of photoelectrons does not depend on the intensity of incident light.

(***ii***) According to wave theory, the light of any frequency can emit electrons from metallic surface provided the intensity of light be sufficient to provide necessary energy for emission of electrons, but according to experimental observations, the light of frequency less than threshold frequency can not emit electrons irrespective of the intensity of incident light.

(***iii***) According to wave theory, the energy transferred by light waves will not go to a particular electron, but it will be distributed uniformly to all electrons present in the illuminated surface.

Therefore, electrons will take some time to collect the necessary energy for their emission.

The time for emission will be more for light of less intensity and vice versa. But experimental observations show that the emission of electrons take place instantaneously after the light is incident on the metal irrespective of the intensity of the light.

Question15:

(*i*) Prove that an ideal capacitor, in an AC circuit does not dissipate power.
(*ii*) Derive an expression for the impedance of an AC circuit consisting of an inductor and a resistor.

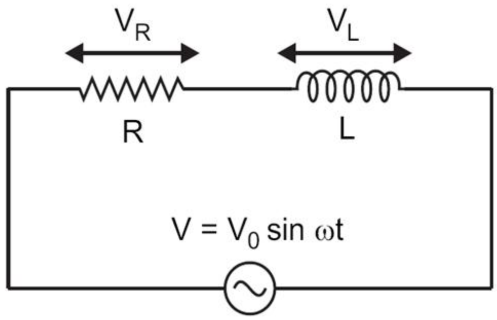
Sol:

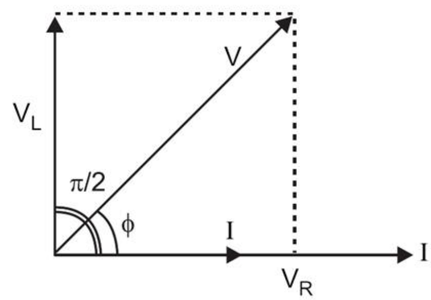
(*i*) Power dissipated in AC circuit is given by, $P = V_{rms} I_{rms} \cos\phi$

For an ideal capacitor $R = 0$. Therefore, $\cos\phi = \frac{R}{Z} = 0$

$\therefore P = V_{rms} I_{rms} \cos 0 = 0.$ Hence, power dissipated in an ideal capacitor is zero.

(*ii*) Suppose a circuit contain a resistor of resistance R and an inductor of inductance L connected in series.
The applied voltage is given by $V = V_o \sin \omega t$. Voltage across resistor is V_R and across inductor is V_L. The voltage V_R and current I are in the same phase, while the voltage V_L leads the current by an angle $\pi/2$. Therefore, voltage V_R and V_L are mutually perpendicular.





The resultant of V_R and V_L is the applied voltage i.e. $V = \sqrt{V_R{}^2 + V_L{}^2}$

We know that, $V_R = i$ R and $V_L = i\,(X_L) = i\,\omega L$.

$\Rightarrow V = \sqrt{(iR)^2 + (iX_L)^2}$

We know that, Impedence = $Z = \dfrac{V}{i}$

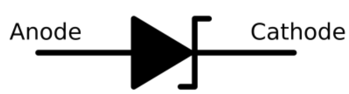
$$\Rightarrow Z = \frac{\sqrt{(iR)^2 + (iX_L)^2}}{i} = \sqrt{R^2 + X_L{}^2} \Rightarrow Z = \sqrt{R^2 + (\omega L)^2}.$$

Question16: With a circuit diagram, explain how a Zener diode can be used as a voltage regulator.

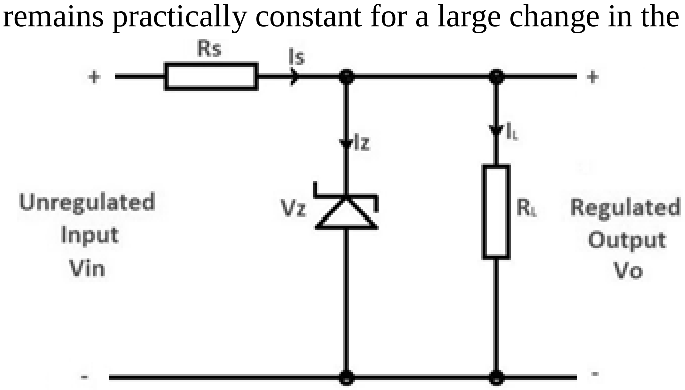
Sol:

Zener diode is a specially designed heavily doped silicon or germanium *p-n* junction diode with high power rating, which can operate continuously, without being damaged in the region of reverse breakdown voltage.

Its symbol is shown in the figure given below:



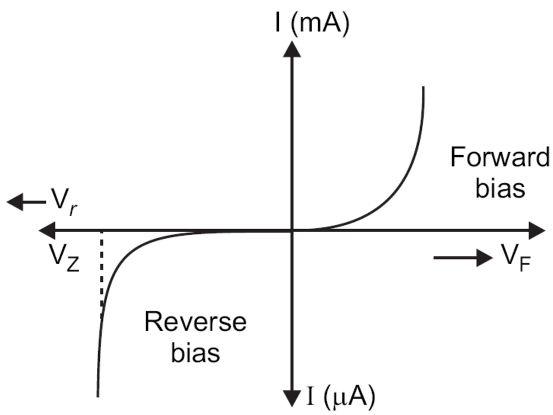
The voltage across it remains practically constant for a large change in the current.



A simple circuit of a voltage regulator using a Zener diode is shown in the figure above. The Zener diode is connected across the load such that it is reverse biased.

When the input DC voltage across the Zener diode increases beyond Zener breakdown voltage , the current through the circuit rises sharply , causing a sufficient increase in the voltage drop across the resistor R_S. As a result, the voltage across the Zener diode remains constant and hence output voltage is raised to normal. So, output voltage remains constant.

I-V Characteristics of Zener Diode are shown in figure given below:



After breakdown voltage V_Z, any variation in current through Zener diode, does not cause any change in Zener voltage. This property of Zener diode is used for regulating supply voltages so that they remain constant.

Question 17: Identify the type of waves which are produced by the following way and write one application for each:
(*i*) Radioactive decay of the nucleus,
(*ii*) Rapid acceleration and decelerations of electrons in aerials,
(*iii*) Bombarding a metal target by high energy electrons.

Answer:

Radioactive decay of the nucleus	Gamma rays	Treatment of tumors
Rapid acceleration and decelerations of electrons in aerials	Radio waves	Radio and television Communication systems
Bombarding a metal target by high energy electrons	X- rays	Study of crystals

Question18: A convex lens made up of glass material having refractive index 1.5 is dipped, in a medium of refractive index 1.65 afterwards in a medium of refractive index 1.33.
(*a*) Will it behave as a converging or a diverging lens in the two cases?
(*b*) How will its focal length change in the two media?
Sol:
(*a*) Let f_{air} be the focal length of lens in air then,

$$\frac{1}{f_{air}}=(n_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f_{air}(n_g-1)}=\frac{2}{f_{air}} \qquad ...(1)$$

<u>Case (*i*):</u> When lens is dipped in a medium A here $n_A=1.65$

Focal length be f_A when dipped in a medium A then, $\frac{1}{f_A}=(n_A-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Using equation (1) we have,

$$\frac{1}{f_A}=\left(\frac{1.5}{1.6(5-1)}\right)\times\frac{2}{f_{air}}=\frac{-1}{5.5f_{air}}$$

$$f_A=-5.5f_{air}$$

As sign of f_A is opposite to that of f_{air},
Therefore, the lens will behave as diverging lens.
<u>Case (*ii*):</u> When lens is dipped in Medium B $n_B=1.33$
Let f_B be the focal length of lens when dipped in medium B.
Then,

$$\frac{1}{f_B}=(n_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{n_g}{n_B}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_B}=\left(\frac{1.5}{1.33}-1\right)\times\frac{2}{f_{air}}=\frac{0.34}{1.33f_{air}}$$

$$f_B=3.91f_{air}$$

As the Sign of f_B is same as that of f_{air} the lens will behave as a converging lens.
(*b*) Change in focal length:
<u>Case (*i*)</u>: Focal length will become negative and its magnitude would increase by half.
<u>Case (*ii*)</u>: Focal length increases.

Question 19: (*i*) Using de Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantization of energy levels in a hydrogen atom.

(*ii*) The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

Sol:

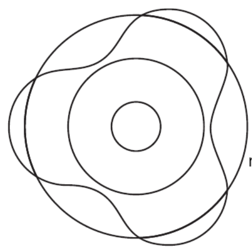
(*i*) From de Broglie's hypothesis, $\lambda = \frac{h}{mv_n} ...(i)$

Where λ is wavelength of electron in moving with velocity v_n in n^{th} orbit.

From de Broglie's condition of stationary orbits, the stationary orbits are those which contain complete de-Broglie wavelength $2\pi r_n = n\lambda ...(ii)$

Substituting value of λ from …(*ii*) in …(*i*), we get $2\pi r_n = n\frac{h}{mv_n} \Rightarrow mv_n r_n = n\frac{h}{2\pi}$.

Which is the Bohr's postulate of quantization of energy levels.



(*ii*) Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{2r} \quad ...(i)$

Potentail energy, $U = -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r} \quad ...(ii)$

Total energy, $E = K + U = -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{2r} \quad ...(iii)$

From equation …(*i*), …(*ii*) and …(*iii*)

$K = -E$ and $U = 2E$

Here, E = –13.6 eV (in ground state)

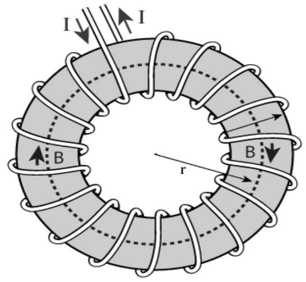
Therefore, Kinetic energy, K = – (–13.6) = 13.6 eV

Potential energy, U = 2 × (– 13.6) = –27.2 eV.

Question20: Using Ampere's circuital law, derive an expression for the magnetic field along the axis of a toroidal solenoid.

Sol:

A toroid is a solenoid of finite length bent in the form of a circle. Let 'N' be the number of turns and I be the current through it.



By Ampere circuital law,

$$\oint \vec{B}.\, d\,\vec{L} = \mu_o NI$$

$$B\oint d\,L = \mu_o NI$$

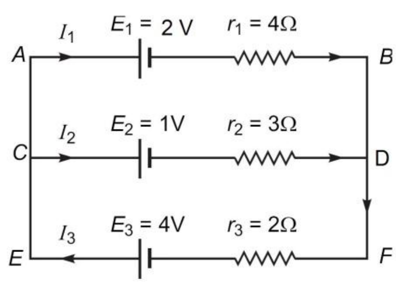
$$B(2\pi r) = \mu_o NI$$

$$B = \frac{\mu_o NI}{2\pi r}$$

$$B = \mu_o nI$$

Where, $n = \dfrac{N}{2\pi r}$ and n is the number of turns per unit length.

Question21: State Kirchhoff's laws. On which principle they are based. Using both laws find the value of currents I_1, I_2 and I_3 in the circuit diagram shown below:



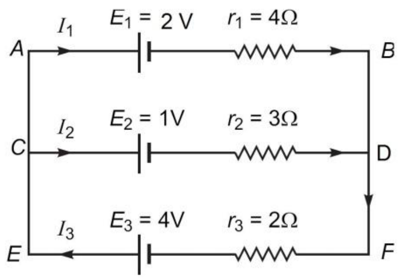
Sol:

Kirchhoff's first law: The algebraic sum of the currents meeting at a junction in a closed electric circuit is zero.

This law is based on the law of conservation of charge.

Kirchhoff's second law: The algebraic sum of changes in potential around any closed path of electric circuit is zero.

This law is based on the conservation of energy.



In the above circuit, by applying Kirchhoff's first law:

$I_3 = I_1 + I_2$ …(*i*)

By applying Kirchhoff's first law in the loop ABDC

$$-2-4I_1+3I_2+1=0$$

$$\Rightarrow 4I_1-3I_2=-1 \text{ ...}(ii)$$

Applying Kirchhoff's second law to loop ABEF

$$-2-4I_1-2I_3+4=0$$
$$\Rightarrow 4I_1+2I_3=2 \text{ or } 2I_1+I_3=1$$

Using …(*i*) we get,

$$\Rightarrow 2I_1+(I_1+I_2)=1$$
$$\Rightarrow \quad 3I_1+I_2=1 \quad \text{...}(iii)$$

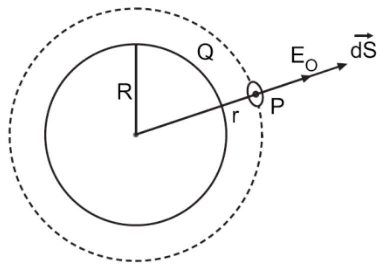
Solving …(*ii*) and …(*iii*), we get

$$I_1=\frac{2}{3}A,\ I_2=\frac{7}{13}A \text{ and } I_3=\frac{9}{13}A.$$

Question22: A thin conducting spherical shell of radius *R* has charge *Q* spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell. Draw a graph of electric field *E*(*r*) with distance *r* from the centre of the shell or $0 \le r \le \infty$.

Sol:

Electric field intensity at a point outside a uniformly charged thin spherical shell: Consider a uniformly charged thin spherical shell of radius R carrying charge Q. To find the electric field outside the shell, we consider a spherical Gaussian surface of radius *r* (> *R*), concentric with given shell.



If $\vec{E}$ is electric field outside the shell, then by symmetry electric field strength has same magnitude E_0 on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E_o}$ and $\vec{ds}$ is zero at each point.

Therefore, electric flux through Gaussian surface $= \oint_S \vec{E_o}.\vec{ds} = \oint_S E_o ds \cos 0 = E_o 4\pi r^2$

Now, Gaussian surface is outside the given charged shell, so charge enclosed by Gaussian surface is Q. Hence, by Gauss's theorem

$$\oint_S \vec{E_o}.\vec{ds} = \frac{1}{\epsilon_o}(\text{Charge enclosed})$$

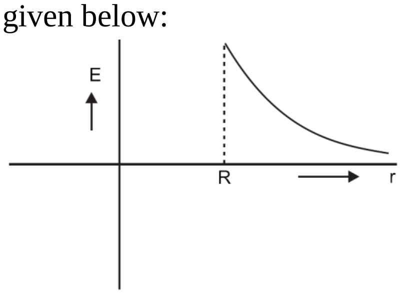
$$\Rightarrow E_o \oint_S ds = \frac{1}{\epsilon_o}(Q) \Rightarrow E_o = \frac{Q}{4\pi \epsilon_o r^2}$$

Thus, electric field outside a charged thin spherical shell is the same as if the whole charge Q is concentrated at the centre.

If σ is the surface charge density of the spherical shell, then, $\phi = 4\pi R^2 \sigma$

$$\Rightarrow E_o = \frac{1}{4\pi \epsilon_o} \frac{4\pi R^2 \sigma}{r^2} = \frac{R^2 \sigma}{\epsilon_o r^2}$$

The graph is shown in the figure given below:



Question 23: Sunita and her friends visited an exhibition. The policeman asked them to pass through a metal detector. Sunita's friends were initially scared of it. Sunita, however, explained to them the purpose and working of the metal detector.

Answer the following questions:

(*a*) On what principle does a metal detector work?

(*b*) Why does the detector emit sound when a person carrying any metallic object walks through it?

(*c*) State some qualities which Sunita displayed while explaining the purpose of walking through the detector.

Answer:

(a) Metal detector works on the principle of resonance in AC circuits.

(b) When a person walks through the gate of a metal detector, the impedance of the circuit changes, resulting in significant change in current in the circuit that causes a sound to be emitted as an alarm.

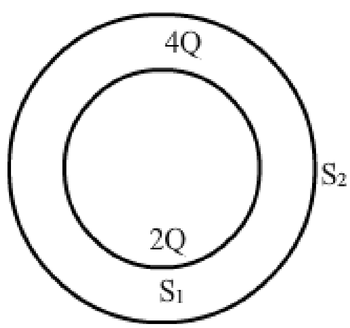
(c) Some of the qualities are: Responsible citizen, knowledgeable person.

Question24: **(*a*)** Find an expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$.

(*b*) Consider two hollow concentric spheres, S_1 and S_2, enclosing charges 2Q and 4Q respectively as shown in the figure.

(*i*) Find out the ratio of the electric flux through them.

(*ii*) How will the electric flux through the sphere S_1 change if a medium of dielectric constant '$\in_r$' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



Sol:

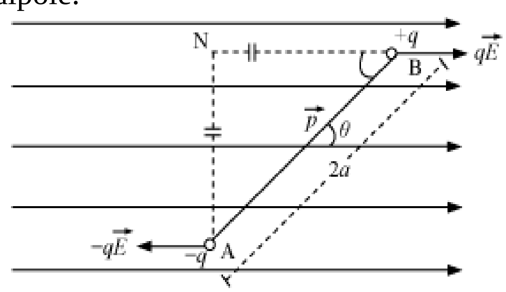
(*a*)Dipole in a Uniform External Field

Consider an electric dipole consisting of charges $-q$ and $+q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with electric field.

Force on charge $-q$ at $A = -q\vec{E}$ (opposite to $\vec{E}$)

Force on charge $+q$ at $B = q\vec{E}$ (along $\vec{E}$)

Electric dipole is under the action of two equal and unlike parallel forces, which give rise to a torque on the dipole.



τ = Force × Perpendicular distance between the two forces

$\tau = qE\,(\mathrm{AN}) = qE\,(2a\sin\theta)$

$\tau = q(2a)\,E\sin\theta$

$\tau = pE\sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E}$

(*b*)

(*i*) Charge enclosed by sphere $S_1 = 2Q$

By Gauss law, electric flux through sphere S_1 is

$$\phi_1 = \frac{2Q}{\epsilon_0}$$

Charge enclosed by sphere $S_2 = 2Q + 4Q = 6Q$

$$\phi_2 = \frac{6Q}{\epsilon_0}$$

The ratio of the electric flux is:

$$\frac{\phi_1}{\phi_2} = \frac{\frac{2Q}{\epsilon_0}}{\frac{6Q}{\epsilon_0}} = \frac{2}{6} = \frac{1}{3}$$

(*ii*) For sphere S_1 with dielectric, the electric flux is:

$$\phi' = \frac{2Q}{\epsilon_r}$$

$$\frac{\phi'}{\phi_1} = \frac{\epsilon_0}{\epsilon_r} \Rightarrow \phi' = \phi_1 . \frac{\epsilon_0}{\epsilon_r}$$

Since, $\epsilon_r > \epsilon_0$

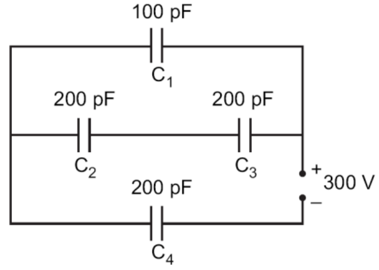
Therefore, $\phi' < \phi_1$

Therefore, the electric flux through the sphere S_1 decreases with the introduction of the dielectric inside it.

OR

Question24: **(*a*)** Derive an expression for the energy stored in a parallel plate capacitor *C*, charged to a potential difference *V*.

(*b*) Obtain the equivalent capacitance of the network given below. For a supply of 300 V, determine the charge and voltage across C_4.



Sol:

(*a*) When a capacitor is charged by a battery, work is done by the charging battery at the expense of its chemical energy. This work is stored in the capacitor in the form of electrostatic potential energy.

Consider a capacitor of capacitance C. Initial charge on capacitor is zero. Initial potential difference between capacitor plates = zero. Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the potential difference between its plates, $V = q/C$

Now work done in giving an additional infinitesimal charge dq to capacitor, $dW = Vdq = \frac{q}{C}dq$

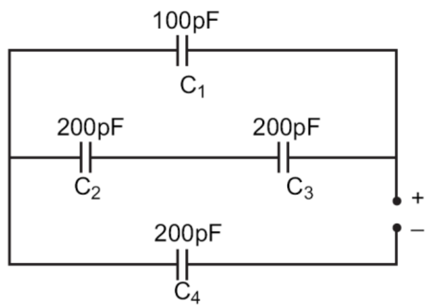
The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore, total work

$$W = \int_0^Q Vdq = \int_0^Q \frac{Q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left[\frac{Q^2}{2} - 0\right] = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then $Q = CV$

$$\Rightarrow W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

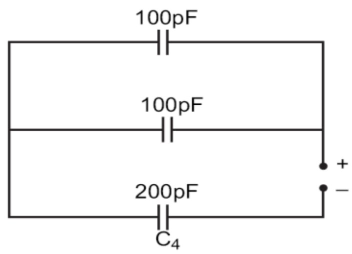
(*b*) In the figure given below:



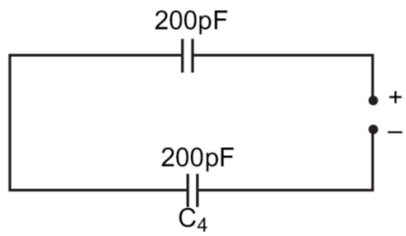
Capacitor C_2 and C_3 are in series, so, their equivalent capacitance will be:

$$\frac{C_2C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100 \text{ pF}$$

Now C_2 and C_3 can be replaced with a new capacitor of 100 pF, as shown in the figure given below,



The two 100 pF capacitor are connected in parallel, so they can be replaced by a new capacitor with capacitance of (100 + 100) pF = 200 pF, as shown in the figure given below,



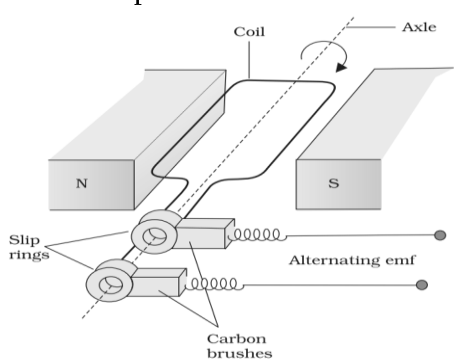
Now, two 200 pF are again connected in parallel, so, their equivalent capacitance is (200 + 200) = 400 pF, as shown in the figure given below

Now, C_{eq} = 100 pF. Now, $Q = C_{eq} \times V = 100 \times 10^{-12} \times 300 = 3 \times 10^{-8}$ Coulomb.
So, Potential difference across $C_4 = Q/C_4 = (3 \times 10^{-8})/(200 \times 10^{-12}) = 150$ V.

Question25. Describe briefly, with the help of a labeled diagram, the basic elements of an AC generator. State its underlying principle. Show diagrammatically how an alternating emf is generated by a loop of wire rotating in a magnetic field. Write the expression for the instantaneous value of the emf induced in the rotating loop.

Answer:

AC generator consists of the four main parts:



(*a*) Field Magnet: It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo. The magnetic field is produced by an electromagnet.
(*b*) Armature: It consists of a large number of turns of insulated wire in the soft iron drum or ring. It can revolve around an axle between the two poles of the field magnet. The drum or ring serves the two purposes: (*i*) It serves as a support to coils and (*ii*) It increases the magnetic field due to air core being replaced by an iron core.
(*c*) Slip Rings: The slip rings R_1 and R_2 are the two metal rings to which the ends of armature coil are connected. These rings are fixed to the shaft which rotates the armature coil so that the rings also rotate along with the armature.

(*d*) Brushes: These are two flexible metal plates or carbon rods (B_1 and B_2) which are fixed and constantly touch the revolving rings. The output current in external load R_L is taken through these brushes.

Principle: When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming's right hand rule. The direction of current remains unchanged during the first half turn of armature. During the second half revolution, the direction of current is reversed. Thus, the direction of induced emf and current changes in the external circuit after each half revolution.
If *N* is the number of turns in coil, *f* the frequency of rotation, *A* area of coil and *B* the magnetic induction, then induced emf

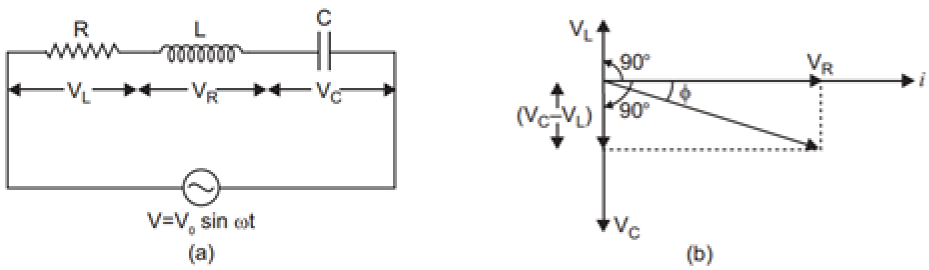
$$e = -\frac{d\phi}{dt} = -\frac{d(N\vec{B}.\vec{A})}{dt} = -\frac{d(NBA\cos\omega t)}{dt} = -\frac{d}{dt}\{NBA(\cos 2\pi ft) = 2\pi NBA \sin 2\pi ft.$$

OR

Question25. Derive an expression for the impedance of a series LCR circuit connected to an AC supply of variable frequency and hence find the expression to find the value of resonant frequency. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set.

Answer:

Suppose resistance *R*, inductance L and capacitance C are connected in series and an alternatingsource of voltage $V = V_0 sin\omega t$ is applied across it. On account of being in series, the current (*i*) flowing through all of them is the same.



Suppose the voltage across resistance R is V_R, voltage across inductance L is V_L and voltage across capacitance C is V_C. The voltage V_R and current i are in the same phase, the voltage V_L will lead the current by angle 90° while the voltage V_C will lag behind the current by angle 90° (figure b). Clearly V_C and V_L are in opposite directions, therefore their resultant potential difference = $V_C - V_L$ ($V_C > V_L$).
Thus V_R and ($V_C - V_L$) are mutually perpendicular and the phase difference between them is 90^o.
As applied voltage across the circuit is V, the resultant of V_R and ($V_C - V_L$) will also be V.
From the figure:

$$V^2 = V_R^2 + (V_C - V_L)^2 \Rightarrow V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

As, $V_R = iR, V_C = iX_C$ and $V_L = iX_L$

Here, $X_C = \text{Capacitive reactance} = \dfrac{1}{\omega C}$

$\therefore V = \sqrt{(iR)^2 + (iX_C - iX_L)^2}$

$\therefore$ Impedence of circuit, $Z = \dfrac{V}{i} = \sqrt{(R)^2 + (X_C - X_L)^2}$

$i.e.\ Z = \sqrt{(R)^2 + (X_C - X_L)^2} = \sqrt{(R)^2 + \left(\dfrac{1}{\omega C} - \omega L\right)^2}$

Instantaneous current, $I = \dfrac{V_o \sin(\omega t + \phi)}{\sqrt{(R)^2 + \left(\dfrac{1}{\omega C} - \omega L\right)^2}}$

Phase difference (ϕ) between current and voltage is given by:

$\tan\phi = \dfrac{X_C - X_L}{R}$

Resonant frequency:

For resonance, $\phi = 0$

$\Rightarrow X_C - X_L = 0$

$\Rightarrow \dfrac{1}{\omega C} = \omega L \Rightarrow \omega^2 = \dfrac{1}{LC}$

$\therefore$ Resonant frequency $= \omega_r = \dfrac{1}{\sqrt{LC}}$

Use of phenomenon of resonance in tuning a radio or TV set:

A radio or a TV set has a L-C circuit with capacitor of variable capacitance C. The circuit remains connected with an aerial coil through the phenomenon of mutual inductance. Suppose a radio or TV station is transmitted a programme at frequency *f*, then waves produce alternating voltage of frequency *f*, in areal, due to which an emf of same frequency is induced in LC circuit, When capacitor C is in circuit is varied then for a particular value of capacitance C,

$f = \dfrac{1}{2\pi\sqrt{LC}}$ the resonance occurs and maximum current flows in the circuit and the TV or radio gets tuned.

Question26. **(*a*)**Derive an expression for constructive and destructive interference in an interference pattern.

(*b*)What is the effect on the interference fringes in a Young's double slit experiment when

(*i*) the separation between the two slits in decreased?

(*ii*) the width of the source slit is increased?

(*iii*) the monochromatic source is replaced by a source of white light?

Justify your answer in each case.

Answer:

(a) Suppose the waves from two coherent sources of light having intensity I_1 and I_2 are represented as

$Y_1 = a\sin\omega t$ and $Y_2 = b\sin(\omega t + \phi)$ where a and b are the respective amplitudes of the two waves and ϕ is the phase angle by which second wave leads the first wave.

By superposition principle, the displacement Y of the resultant wave at time (t) would be given by

$$Y_1 + Y_2 = a\sin\omega t + b\sin(\omega t + \phi) = a\sin\omega t + b\sin\omega t\cos\phi + b\cos\omega t\sin\phi$$

$$\Rightarrow Y = \sin\omega t(a + b\cos\phi) + \cos\omega t\, b\sin\phi \qquad ...(i)$$

Now, let $a + b\cos\phi = R\cos\theta \ldots(ii)$ and $b\sin\phi = R\sin\theta \ldots(iii)$

So, the equation …(i) can be rewritten as

$$Y = R[\sin\omega t\cos\theta + \cos\omega t\sin\theta] \Rightarrow Y = R\sin(\omega t + \phi)$$

Squaring equation…(i) , …(ii) and adding, we get

$$R^2(\cos^2\theta + \sin^2\theta) = (a + b\cos\phi)^2 + (b\sin\phi)^2$$

$$\Rightarrow R^2 = a^2 + b^2\cos^2\phi + 2ab\cos\phi + b^2\sin^2\phi = a^2 + b^2 + 2ab\cos\phi$$

$$\Rightarrow R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

Therefore, the resultant wave is a harmonic wave of amplitude R.

We know that intensity of a wave is directly proportional to the square of the amplitude of the wave.

Therefore, $I_1 = k\,a^2$ and $I_2 = k\,b^2$ and amplitude of resultant wave be I_R is

$$I_R = KR^2 = K\left(a^2 + b^2 + 2ab\cos\phi\right)$$

$$\Rightarrow I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Now, for constructive interference I_R should be maximum, which will be possible if $\cos\phi = \max = +1$

$\therefore \phi = 0, 2\pi, 4\pi \ldots$ or $\phi = 2n\pi$ where $n = 0, 1, 2, 3 \ldots$

For destructive interference I_R sould be minimum, which will be possible if $\cos\phi = \min = -1$

$\therefore \phi = \pi, 3\pi, 5\pi \ldots$ or $\phi = (2n - 1)\pi$ where $n = 1, 2, 3 \ldots$

(b) (i)The fringe width β in case of interference pattern in case of interference pattern is given by

$$\beta = \frac{D\lambda}{d}$$

Where, D is the distance between slits and the screen, d is the separation between the slits and λ is the wavelength of the light used.

As the separation between the two slits (D) is decreased, then the fringe width (β) will also decrease.

(ii)Suppose s is the size of the source slit and S is the distance of the of the source from the plane of two slits, then the condition for the interference fringes to be seen is $\frac{s}{S} < \frac{\lambda}{D}$.

If the source slit is brought closer to double plane of the slits, then S decreases due to which interference pattern gets less and less sharp. If the source will be brought too close then the fringes will get disappear.

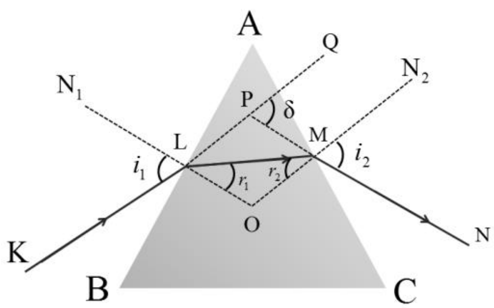
(iii) If the source of white light is used instead of monochromatic source then, the central fringe will be white. Also the blue colour has lowest wavelength, so the fringe closest to the both sides of white fringe will be of blue colour and the farthest fringe will be of red colour. After few colour fringes, interference pattern will get disappear.

OR

Question26. Derive an expression for the Prism formula? Find the angle of deviation produced by a prism of angle 9^o, given refractive index of the material is 1.6.

Answer:

Suppose a ray is light is incident on the prism and after two refractions it deviates from its path as shown in the figure given below.



In the figure shown above, ABC is the principal section of prism with angle A, KL is the incident ray LM is the first refracted ray and MN is the second refracted ray.

N_1 and N_2 are normal to the surfaces of the prism AB and AC respectively. Angle of incident and angle of refraction are at the two faces is also shown in the figure.

Here, deviation of the light ray $= \angle QPN = \delta$.

From ΔPLM, $\delta = \angle PLM + \angle PML$

$$\Rightarrow \delta = (\angle i_1 - \angle r_1) + (\angle i_2 - \angle r_2)$$

$$\Rightarrow \delta = (\angle i_1 + \angle i_2) - (\angle r_1 + \angle r_2) \quad ...(i)$$

Now in ΔOLM, $\angle O + \angle r_1 + \angle r_2 = 180^o \quad ...(ii)$

Also, $\angle ALO + \angle AMO = 180^o$

As $\angle ALO$, $\angle AMO$, $\angle A$ and $\angle O$ are the angles of a cyclic quardilateral

$\Rightarrow \angle A + \angle O = 180^o \quad ...(iii)$

From equation ...(ii) and ...(iii)

$\angle O + \angle r_1 + \angle r_2 = \angle A + \angle O$

$\Rightarrow \angle A = \angle r_1 + \angle r_2$ ....(iv)

From equation ...(i) and ...(iv)

$\delta = (\angle i_1 + \angle i_2) - \angle A$ ...(v)

Rewriting equation …(v),

$\delta = (\sqrt{\angle i_1})^2 + (\sqrt{\angle i_2})^2 - \text{A}$

$\Rightarrow \delta = (\sqrt{\angle i_1})^2 + (\sqrt{\angle i_2})^2 - 2\sqrt{i_1 i_2} + 2\sqrt{i_1 i_2} - \text{A}$

$\Rightarrow \delta = \left(\sqrt{\angle i_1} - \sqrt{\angle i_2}\right)^2 + 2\sqrt{i_1 i_2} - \text{A}$

Therefore, value of δ will be minimum if $\sqrt{\angle i_1} = \sqrt{\angle i_2}$ or $\angle i_1 = \angle i_2$ also in that case $\angle r_1 = \angle r_2$

Let δ_m is the angle of minimum deviation, $\angle i_1 = \angle i_2 = \angle i$ and $\angle r_1 = \angle r_2 = r$

From equation ...(iv) $\angle r = \dfrac{\angle A}{2}$

From equation ...(i) $\delta_m = \angle i + \angle i - A$ or $\angle i = \dfrac{\angle A + \delta_m}{2}$

If μ is the refractive index of the material of the prism, then according to the Snell's law,

$$\mu = \frac{\sin i}{\sin r} \text{ or } \mu = \frac{\sin(A+\delta_m)/2}{\sin A/2}.$$

Given, angle of prism $= A = 9^{\circ}$, refractive index of material of prism $= \mu = 1.6$

If the angle of the prism is less than 10°, then the given prism is a thin prism and deviation produced by this prism is given by $\delta = (\mu - 1)A$, where symbols have their usual meaning.

$\therefore \delta = (\mu - 1)A \Rightarrow \delta = (1.6 - 1)9 = 5.4^{\circ}$.