Section - D

30) Frequency distribution:

(choice 1)

4

Class	Frequency	x_i	f_ix_i
11-13	3	12	$3\times12=36$
13-15	6	14	$6\times14=84$
15-17	9	16	$9\times16=144$
17-19	13	18	$13\times18=234$
19-21	f	20	$f\times20=20f$
21-23	5	22	$5\times22=110$
23-25	4	24	$4\times24=96.$
Total: →	$40+f$		$704+20f$

Given, mean = 18. To find: Value of f.

We know,

$$\text{mean } (\bar{x}) = \frac{\sum f_ix_i}{\sum f_i}$$

$$\Rightarrow 18 = \frac{704+20f}{40+f}$$

$720+18f = 704+20f.$

$720-704 = 20f-18f$

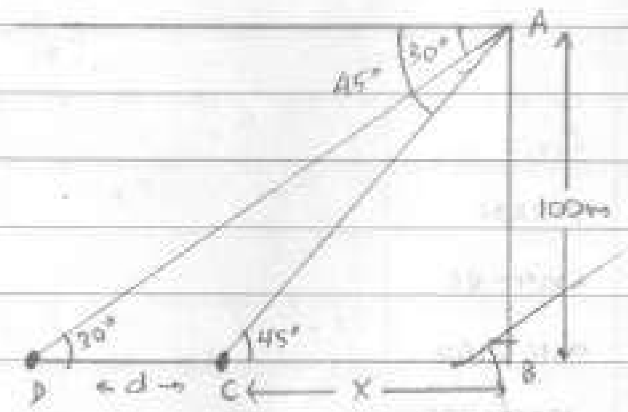
$16 = 2f$

$\Rightarrow f = 8.$

$\boxed{\text{The value of } f \text{ is } 8}$

29) Diagram:

4



AB → lighthouse = 100m high

C → boat 1

D → boat 2.

To find: $\overline{CD}$ or d.

(distance b/w ships)

We know,

$\tan\angle ACB = \frac{opp.}{adj.} = \frac{AB}{BC}.$

$\rightarrow \tan 45° = \frac{100}{x}$

$1 = \frac{100}{x}$

$\Rightarrow x = 100m.$

$\tan\angle ADB = \frac{opp.}{adj.} = \frac{AB}{BD}.$

$\rightarrow \tan 30 = \frac{100}{x+d}$

$\frac{1}{\sqrt{3}} = \frac{100}{d+100}$ $[x=100]$

$100 + d = 100\sqrt{3}.$

$\rightarrow d = 100\sqrt{3} - 100. = 100(\sqrt{3}-1)$

Given, $\sqrt{3} = 1.732,$

$\Rightarrow d = 100(1.732-1)$

$= 100 \times 0.732 = 73.2m.$

$\rightarrow$ $\boxed{\text{The distance between the boats is } 73.2m.}$

27) To prove: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

4

Simplifying LHS:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$$

$$= \frac{\sin A}{\cos A}\left[\frac{1 - (2\sin^2 A)}{2\cos^2 A - 1}\right]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - (\sin^2 A + \cos^2 A)}\right] \qquad \left[\because \sin^2 A + \cos^2 A = 1\right]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right]$$

$$= \frac{\sin A}{\cos A} \times 1$$

$$= \tan A. \qquad \left[\because \frac{\sin A}{\cos A} = \tan A\right]$$

LHS = RHS

hence proved.

28) i) Given, metal bucket shaped like frustum of cone.

Dimensions: height = 24 cm. (h)

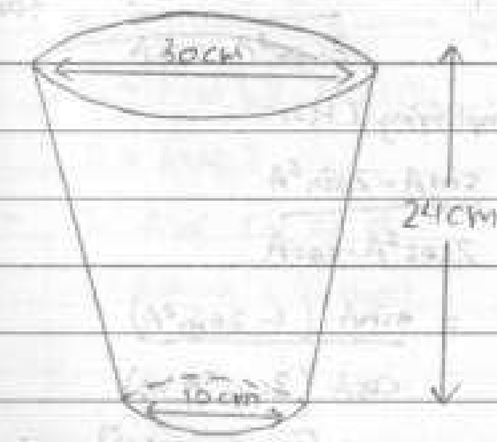
lower diameter = 10 cm → radius = $\frac{10}{2}$ = 5 cm (r_1)

upper diameter = 30 cm → radius = $\frac{30}{2}$ = 15 cm (r_2)

To find: Metal sheet needed to make bucket.

→ Area of metal sheet = Curved surface area + Area of base.

Bucket



We know,

Curved surface area of frustum = $\pi \times (R+r) \times l$ sq. units.

where $l = \sqrt{h^2 + (R-r)^2}$ units.

To find l: $l = \sqrt{h^2 + (R-r)^2}$ units.

$= \sqrt{24^2 + (15-5)^2}$ cm

$= \sqrt{576 + 10^2}$

$= \sqrt{676}$

$l = 26$ cm.

⇒ CSA = $\pi (R+r) l$ sq. units

$= 3.14 \times (15+5) \times 26$ sq. cm

$= 3.14 \times 20 \times 26$

CSA = 1632.8 cm^2

We know, Area of circular base = πr^2 sq. units.

$\rightarrow$ Area = $3.14 \times 5 \times 5$ sq.cm

$= \frac{3.14 \times 100}{4}$

$= \frac{314}{4}$

Area $= 78.5$ cm^2.

Total area of sheet = Curved area + Circular base area

$= 1632.8 + 78.5$ cm^2

Area $= 1711.3$ cm^2

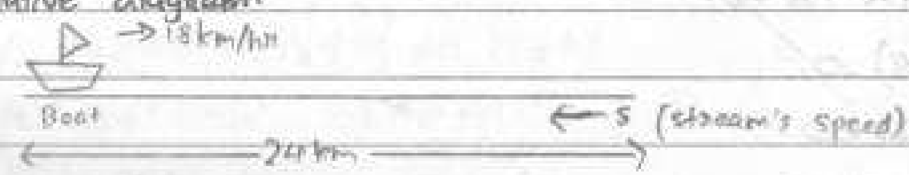
$\boxed{\text{The area of sheet needed is } 1711.3 \ cm^2}$

(ii) Plastic buckets are less preferable to metal buckets because plastic buckets are more harmful to the environment. They may also leak harmful chemicals into the water being stored.

23) Representative diagram:

(choice 1)

4



Given that:

Speed of boat = 18 km/hr in still water.

Speed of stream = s (variable, must find)

Distance upstream + back = 24 km

Time upstream = 1 hr more than time downstream.

We know, $\text{speed} = \frac{\text{Distance}}{\text{Time}} \rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\text{Time upstream} = \frac{24}{18-s}$, $\text{Time downstream} = \frac{24}{18+s}$.

$\rightarrow \frac{24}{18-s} = 1 + \frac{24}{18+s}$.

$\frac{24}{18-s} = \frac{18+s+24}{18+s}$. (cross-multiplying)

$24(18+s) = (42+s)(18-s)$

$432+24s = 756+18s-42s-s^2$

$\rightarrow s^2+24s+24s+432-756=0$.

$s^2+48s+(-324)=0$.

$s^2+54s-6s-324=0$.

$s(s+54)-6(s+54)=0$.

$(s-6)(s+54)=0$.

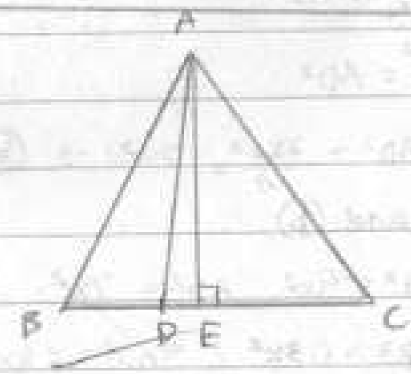
Now, either $s-6=0$ or $s+54=0$.

$\rightarrow s=6$ $\rightarrow s=-54$.

So speed $= 6$ or -54 km/hr.

But speed cannot be negative.

$\Rightarrow$ $\boxed{\text{Speed of the stream is 6 km/hr.}}$



25) Given: $\triangle ABC$ is equilateral.

(4)

$\rightarrow AB = BC = CA$, $\angle A = \angle B = \angle C = 60°$

D is a point on BC such that $BD = \frac{1}{2} BC$.

To prove: $9(AD)^2 = 7(AB)^2$.

Construction: Draw $AE \perp BC$.

Proof: Let $BD = x$.

$\Rightarrow BC = 3x = AB = AC$ [$\because \triangle ABC$ is equilateral] [Given $BD = \frac{1}{3}BC$].

Also, we know that $BE = \frac{1}{2}BC$ [Altitude in equilateral $\triangle$ bisects base].

As $\angle AEB = 90°$,

In ~~triangle~~ $\triangle ABE$, by Pythagoras Theorem,

$BE^2 + AE^2 = AB^2$. $\rightarrow AB^2 = 9x^2$. $\rightarrow$ (2).

$\left(\frac{3x}{2}\right)^2 + AE^2 = (3x)^2$.

$\frac{9x^2}{4} + AE^2 = 9x^2$ $\rightarrow AE^2 = \frac{3 \times 9x^2}{4}$ $\rightarrow$ (1)

Now, in $\triangle ADE$, $\angle E = 90°$.

By Pythagoras Theorem,

$DE^2 + AE^2 = AD^2$

$\left(\frac{3x}{2} - x\right)^2 + \frac{27x^2}{4} = AD^2$ [From ①].

$\left(\frac{x}{2}\right)^2 + \frac{27x^2}{4} = AD^2$.

$\frac{x^2 + 27x^2}{4} = AD^2$

$\Rightarrow AD^2 = \frac{28x^2}{4} = 7x^2 \rightarrow$ ⓑ.

$DE = BE - BD$

$= \frac{3x}{2} - x$.

From ⓐ and ⓑ,

$AB^2 = 9x^2$, $AD^2 = 7x^2$.

$7AB^2 = 63x^2$, $9AD^2 = 63x^2$.

$\Rightarrow 7AB^2 = 9AD^2$.

hence proved.

24) Given, sum of 4 consecutive no. in AP is 32.

Also, ratio of $t_1 \times t_n$ (first × last) and two middle terms product = 7 : 15.

Let no. of terms be n. nth term = $t_n = a + (n-1)d$.

$\Rightarrow t_1 = a + (1-1)d$.

$t_n = a + (n-1)d$.

As there are two middle terms, n is even.

$\Rightarrow$ middle term 1 = $a + \left(\frac{n+2}{2} - 1\right) d$. Let this be α.

middle term 2 = $a + \left(\frac{n}{2} - 1\right) d$. Let this be β.

Given, $\frac{t_1 \cdot t_n}{\alpha \cdot \beta} = \frac{7}{15}$.

$$\frac{a \times [a + (n-1)d]}{\left(a + \frac{n}{2}d\right)\left[a + \left(\frac{n}{2} - 1\right)d\right]} = \frac{7}{15}.$$

$$\Rightarrow \frac{a^2 + a(n-1)d}{a^2 + \frac{n}{2}d \cdot \left(\frac{n}{2} - 1\right)d + a\left[\frac{n}{2}d + \left(\frac{n}{2} - 1\right)d\right]} = \frac{7}{15}.$$

$$\frac{a^2 + a(n-1)d}{a^2 + \frac{n^2}{4}d^2 - \frac{n}{2}d^2 + a(n-1)d} = \frac{7}{15}.$$

Cross-multiplying,

$$15a^2 + 15a(n-1)d = 7a^2 + \left(\frac{7n^2}{4} - \frac{7n}{2}\right)d^2 + 7a(n-1)d.$$

$$8a^2 + 8a(n-1)d = \left(\frac{7n^2}{4} - \frac{7n}{2}\right)d^2.$$

Assuming we take only 4 terms, then n will be 4.

Substituting n=4 in above equation,

$8a^2 + 8a(4-1)d = \left(\frac{7\cdot 4^2}{4} - \frac{7\times 4}{2}\right)d^2$

$8a^2 + 24ad = (28-14)d^2$

$8a^2 + 24ad = 14d^2 \rightarrow ①$.

Sum = 32. $\rightarrow S_n = \frac{n}{2}[2a+(n-1)d]$.

$32 = \frac{4}{2}[2a+3d]$.

$2a+3d = 16 \rightarrow ②$.

Squaring ②, $4a^2 + 9d^2 + 12ad = 256 \rightarrow$ (2.1)

Substituting ③ in (2.1),

$-15d^2 + 9d^2 = 256$

$-6d^2 = 256$

$d^2 = \frac{256}{6}$

Substituting ③ in (2.1), $7d^2 + 9d^2 = 256$.

$16d^2 = 256 \rightarrow d^2 = 16,\ d = 4$.

Now, $2a + 3(4) = 16$.

$2a = 4$

$a = 2$. $\rightarrow$ Terms = 2, 6, 10, 14.

a and a+2d a+3d.

$\boxed{\text{The numbers are } 2, 6, 10 \text{ and } 14.}$

When n=4,

~~terms~~ $a+d,\ a,\ a+2d, a+3d$.

$7a(a+3d) = (a+d)(a+2d)\times 15$.

$7(a^2+3ad) = (a^2+ad+2d^2+2ad)15$

$\therefore \cancel{2d^2=0}$.

$\cancel{d=0}$.

$7a^2+21ad = 15a^2+45ad+30d^2$

$-30d^2 = 8a^2+24ad$

$-15d^2 = 4a^2+12ad \rightarrow ③$.

terms a+d, a, a+2d, a+3d.

$\frac{a(a+3d)}{(a+2d)(a+d)} = \frac{7}{15}$. (Given)

$15a^2 + 45ad = 7[a^2+3ad+2d^2]$

$15a^2 + 45ad = 7a^2 + 21ad + 14d^2$.

$8a^2 - 14d^2 + 24ad = 0$.

$14d^2 = 8a^2 + 24ad$.

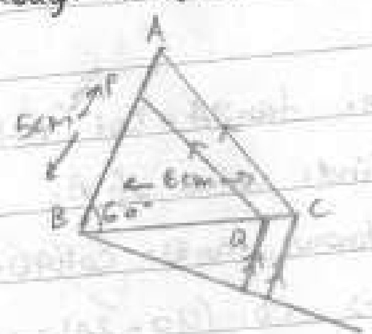
$7d^2 = 4a^2 + 12ad \rightarrow ③$.

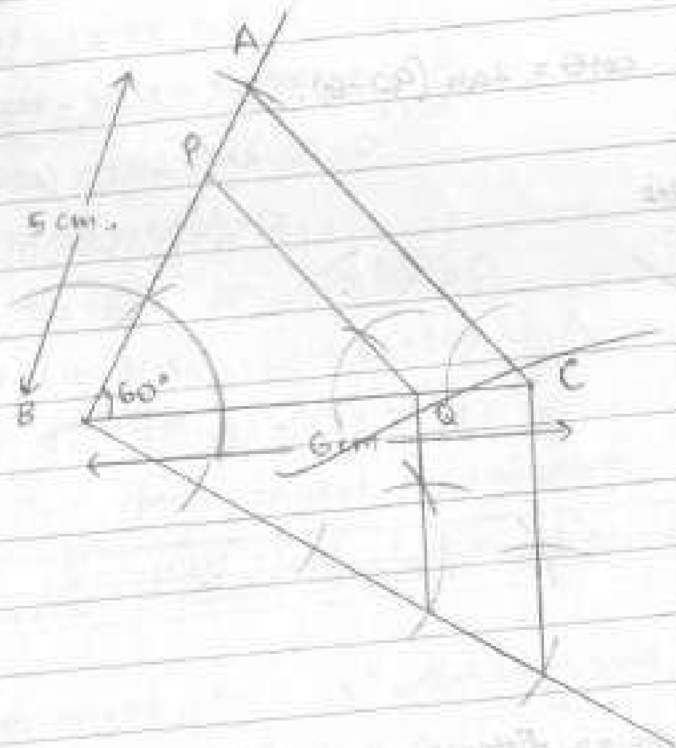
26) Given: $\triangle ABC$, $BC = 6cm$, $AB = 5cm$, $\angle ABC = 60°$.

To draw: $\triangle$ w/ $\frac{3}{4}$ sides of $\triangle ABC$.

4

Rough Diagram.





$AB = 5cm$

$BC = 6cm$

$\angle ABC = 60°$

$\triangle PBQ$ is required triangle.

$PB = \cancel{B}\ \cancel{3}.75$ cm.

$BQ = 4.5$ cm.

$PQ = 4.25$ cm.

Section-c

19) Given: $\tan 2A = \cot(A-18)$, $0 \leq 2A < 90°$. ($2A$ is acute)

(Choice 2) To find: value of A.

3

We know, $\tan\theta = \cot(90-\theta)$ and $\cot\theta = \tan(90-\theta)$.

$\rightarrow \cot(90-2A) = \cot(A-18)$

Applying $\cot^{-1}$, on both sides

$90 - 2A = A - 18$.

$108 = 3A$.

$\rightarrow A = 36°$.

$\boxed{\text{The value of } A \text{ is } 36°.}$

16) Given: distance is 1500 km.

Usual speed = s.

3

We know, speed = $\frac{\text{distance}}{\text{time}}$ $\rightarrow$ ~~[illegible]~~ time = $\frac{\text{distance}}{\text{speed}}$.

$\rightarrow$ From question, $\frac{1500}{100+s} + \frac{1}{2} = \frac{1500}{s}$. [half an hour late].

(30 mins = 0.5 hr).

$\frac{1500}{100+s} = \frac{1500}{s} - \frac{1}{2}$.

$\frac{1500}{100+s} = \frac{3000-s}{2s}$. Cross multiplying,

$3000s = 300000 - 100s + 3000s - s^2$.

$s^2 + 100s - 300000 = 0$.

$s^2 + 600s - 500s - 300000 = 0$.

$s(s+600) - 500(s+600) = 0$

$(s-500)(s+600) = 0$.

$\Rightarrow s - 500 = 0$ or $s + 600 = 0$.

$\rightarrow s = 500$ km/h $\rightarrow s = -600$ km/h.

$\Rightarrow s = 500$ or -600 km/hr.

But speed cannot be negative.

$\Rightarrow$ $\boxed{\text{The usual speed of the plane is 500 km/hr.}}$

4) Given, polynomial $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$.

3

Two zeroes $\rightarrow 2+\sqrt{3}$ and $2-\sqrt{3}$.

$\Rightarrow$ Product of two zeroes is also a zero.

$\Rightarrow (2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = 1$. $[(a+b)(a-b) = a^2 - b^2]$.

As 1 is a zero, $\Rightarrow x-1$ is a factor.

Dividing,

$$x-1 \overline{)\, 2x^4 - 9x^3 + 5x^2 + 3x - 1 \,}\left(2x^3 - 7x^2 - 2x + 1\right.$$

$$\underline{2x^4 - 2x^3}$$

$$-7x^3 + 5x^2$$

$$\underline{-7x^3 + 7x^2}$$

$$-2x^2 + 3x$$

$$\underline{-2x^2 + 2x}$$

$$x - 1$$

$$\underline{x - 1}$$

$$0$$

=> By division algorithm,

$$p(x) = (x-1)\underbrace{(2x^3 - 7x^2 - 2x + 1)}_{g(x)}.$$

Now, in a cubic polynomial, we know:

$$\text{sum of roots} = \frac{-\text{coeff. of } x^2}{\text{coeff. of } x^3}.$$

The roots of $g(x)$ are $2+\sqrt{3}$, $2-\sqrt{3}$ and α.

$$\rightarrow \alpha + 2 + \sqrt{3} + 2 - \sqrt{3} = \frac{-(-7)}{2}$$

$$\alpha + 4 = \frac{7}{2}$$

$$\alpha = -\frac{1}{2}. \text{ which is hence a zero of } p(x)$$

=> $\boxed{\text{All zeroes are } -\frac{1}{2},\ 1,\ 2+\sqrt{3} \text{ and } 2-\sqrt{3}.}$

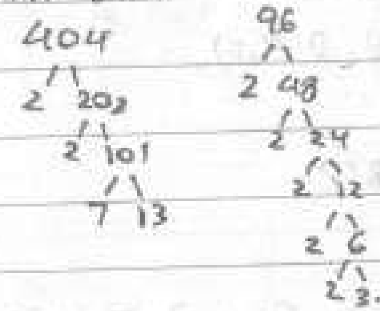
06/8/024

(3) Numbers: 404, 96. To find: HCF and LCM.

3

$$\begin{array}{r|l} 2 & 404, 96 \\ \hline 2 & 202, 48 \\ \hline & 101, 24 \end{array}$$

$\Rightarrow$ Then HCF is 4.



$404 = 2^2 \times 7 \times 13$

$96 = 2^5 \times 3$

$\boxed{HCF = \text{greatest common factor} = 2^2 = 4.}$

LCM = all factors (least power) = $2^5 \times 3 \times 7 \times 13$

$= 96 \times 101$

$= \boxed{9696}$

Product of two numbers = 96×404

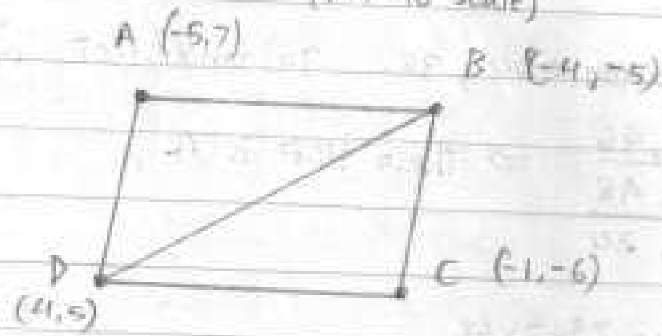
$= 38784$

Product of HCF & LCM = 9696×4

$= 38784$.

Hence, HCF × LCM = product of two numbers.

(not to scale)



15) Vertices of quadrilateral ABCD:

(choice 2) A (-5,7), B(-4,-5), C (-1,-6), D(4,5)

3

Area of quad ABCD.

= area $\triangle ABD$ + area $\triangle BCD$.

area $\triangle ABD \rightarrow$

$= \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$ sq. units.

$= \frac{1}{2}\left[-5(-5-5)+(-4)(5-7)+4(7+5)\right]$

$= \frac{1}{2}\left[\cancel{\#} \; 50+8+48\right]$

$= \frac{1}{2}\left[58+48\right]$.

$= \frac{1}{2} \times 106 = 53$ sq. units^2.

area $\triangle BCD = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$.

$= \frac{1}{2}\left[-4(-6-5)+(-1)(5+5)+(4)\overset{(-5+6)}{\cancel{(\ldots)}}\right]$.

$= \frac{1}{2}\left[44-10+4\right]$

$= \frac{1}{2} \times \cancel{38} = 19$ sq. units^2.

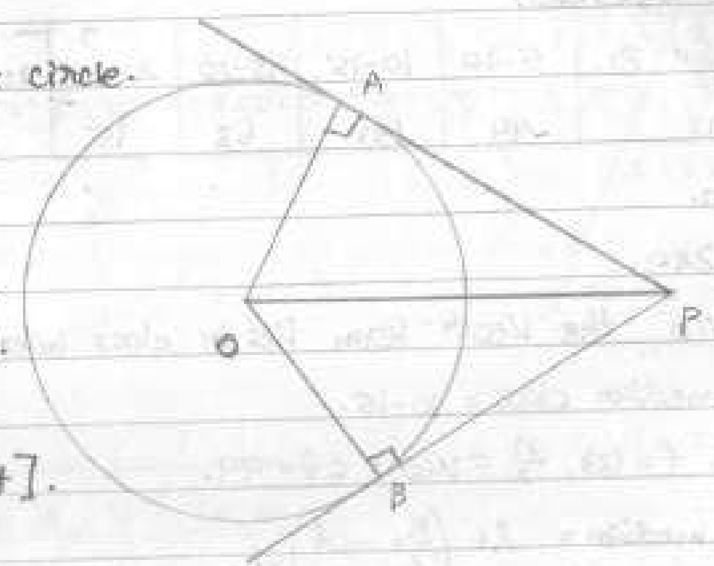
$\Rightarrow$ Area of quadrilateral = Area of two triangles = $53+19 = 72$ sq. units.

$\boxed{\text{Area of quadrilateral ABCD is 72 sq. units}}$

(18) Given: Circle (O, r). AP and PB are tangents drawn to the circle.

3

To prove: PA = PB.



Construction: Join OA, OB and OP.

Proof: OA = OB [radius]. (side).

$\angle OAP = \angle OBP = 90°$ (right angle).

[∵ radius is perpendicular to tangent at point of contact].

OP = OP (hypotenuse).

So in $\triangle OAP$ and $\triangle OBP$,

by RHS congruency,

→ $\triangle OAP \cong \triangle OBP$.

by CPCT,

⇒ AP = BP.

hence proved.

22) Distribution of frequencies:

3

Salary in thousand Rs.	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of persons	49	133	63	15	6	7	4	2	1

To find: median.

no. of people = 280.

$\Rightarrow \frac{n}{2} = 140$, the 140^{th} term lies in class interval 10-15.

$\Rightarrow$ median class = 10-15.

$l = 10,\ h = 5,\ f = 133,\ \frac{n}{2} = 140,\ cf = 49.$

We know, median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h.$

$$\Rightarrow \text{median} = 10 + \frac{140 - 49}{133} \times 5.$$

$$= 10 + \frac{\cancel{91}^{13}}{\cancel{133}_{19}} \times 5.$$

$$= 10 + \frac{65}{19}$$

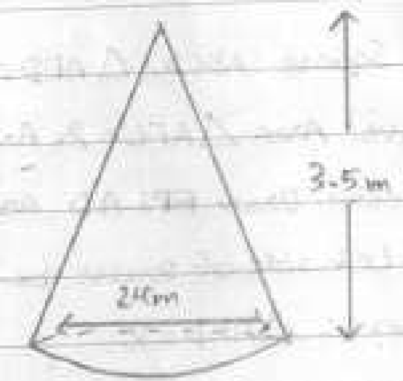
$$= 10 + 3.421$$

$$= 13.421.$$

$\boxed{\text{The median salary is 13.421 thousand rupees.}}$

21) Conical heap of rice:

(Choice 2) Dimensions: diameter = 24m, height 3.5 m. → radius = 12m.



Volume of cone $= \frac{1}{3} \times \pi r^2 h$ cu. units.

$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$ cu. m.

$= 132 \times 4$

$= 528$ cu. m.

$\boxed{\text{The volume of the rice heap is 528 cu. m.}}$

Area of cloth required = Curved surface area.

CSA of cone $= \pi r l$ sq. units where $l = \sqrt{h^2 + r^2}$ units.

Finding l: $l = \sqrt{h^2 + r^2}$ units

$= \sqrt{3.5^2 + 12^2}$ m

$= \sqrt{12.25 + 144}$

$= \sqrt{156.25}$

$= 12.5$ m.

$\Rightarrow$ CSA $= \pi r l$ sq. units

$= \frac{22}{7} \times 12.5 \times 12$

$= \frac{22 \times 150}{7} = \frac{3300}{7} = 471.\overline{428571}\ m^2.$

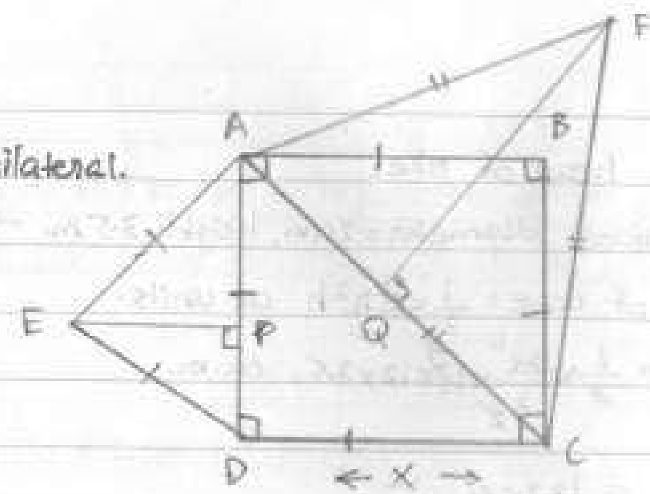
$\boxed{\text{The area of canvas cloth required is } 471.\overline{428571}\ m^2.}$

17) Given: Square ABCD. $\triangle AED$ and $\triangle AFC$ are equilateral.

(choice 1) To prove: Area $\triangle AFC = 2 \times$ Area $\triangle AED$.

3

Construction: Draw $EP \perp AD$ and $FQ \perp AC$.



Proof: Let side of square be x.

$\Rightarrow$ sides of $\triangle AED = x$.

In $\triangle ABC$, $\angle B = 90°$.

$\Rightarrow$ By Pythagoras Theorem,

$AB^2 + BC^2 = AC^2$

$x^2 + x^2 = AC^2 \Rightarrow AC = \sqrt{2}x$. $\Rightarrow$ sides of $\triangle AFC = \sqrt{2}x$.

We know, altitude of equilateral $\triangle$ bisects the base.

$\rightarrow PD = \frac{x}{2}$, $AQ = \frac{x}{\sqrt{2}}$.

In $\triangle AEP$, $\angle P = 90°$.

By Pythagoras theorem, $AE^2 = EP^2 + AP^2$.

$$x^2 = EP^2 + \left(\frac{x}{2}\right)^2.$$

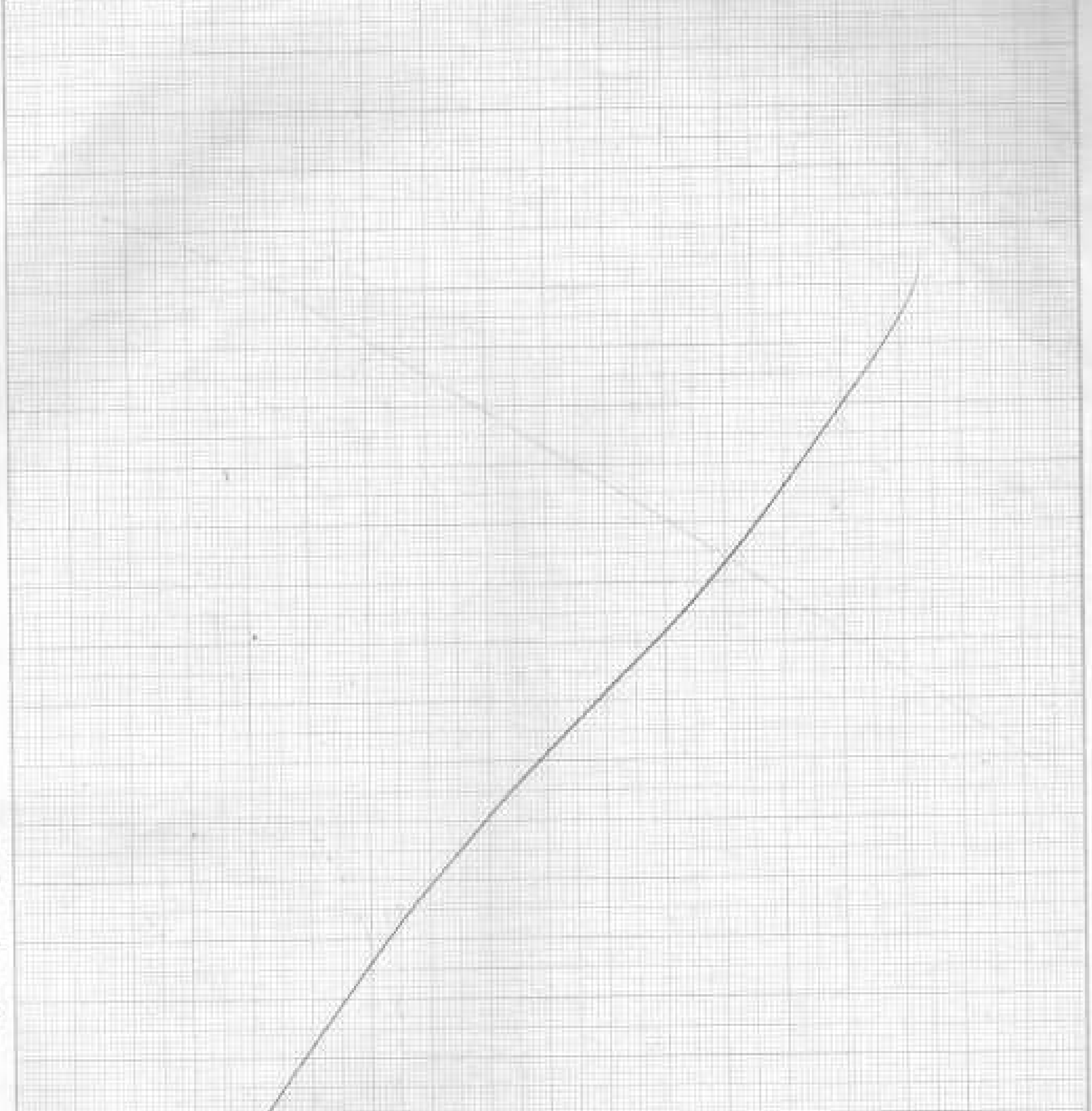
$$EP^2 = \frac{3x^2}{4} \Rightarrow EP = \frac{\sqrt{3}}{2}x^2.$$

In $\triangle AFQ$, $\angle Q = 90°$.

By Pythagoras theorem, $AF^2 = FQ^2 + AQ^2$

$$2x^2 = FQ^2 + \frac{x^2}{2}.$$

$$FQ^2 = \frac{3x^2}{2} \Rightarrow FQ = \frac{\sqrt{3}x}{\sqrt{2}}.$$



We know, Area of triangle $= \frac{1}{2} \times$ Base $\times$ height sq. units.

$\Rightarrow$ Area of $\triangle AFC = \frac{1}{2} \times \sqrt{2}x \times FA$

$= \frac{1}{2} \cdot \sqrt{2}x \cdot \frac{\sqrt{3}}{\sqrt{2}}x$

$= \frac{\sqrt{3}}{2}x^2.$

Area of $\triangle AED = \frac{1}{2} \times x \times EP.$

$= \frac{1}{2} \cdot x \cdot \frac{\sqrt{3}}{2}x$

$= \frac{\sqrt{3}}{4}x^2.$

$\therefore$ Area of $\triangle AED = \frac{\sqrt{3}}{2}x^2 =$ Area of $\triangle AFC$.

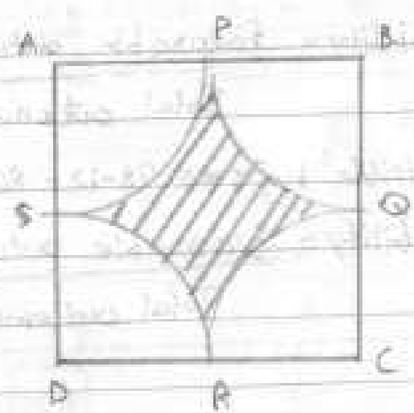
hence proved.

20) Given: side of square ABCD = 12 cm.

3

To find: shaded area.

Shaded area + Area of 4 quadrants = Area of square.



Area of square $= s^2$ sq. units

$= 12^2 = \cancel{144}$ cm^2.

Area of quadrant $= \frac{1}{4} \times \pi r^2$ sq. units

$= \frac{1}{4} \times 3.14 \times \frac{\cancel{12}^{2}}{2} \times \frac{\cancel{12}^{3}}{2}$

$= 9 \times 3.14 = \cancel{26.26}$ 28.26 cm^2.

=> shaded area = Area of square − 4×(Area of quadrant) sq.units

$= 144 - 4(28.26)$ sq.cm

$= 144 - 113.04$

$= 30.96\ cm^2.$

$\boxed{\text{The area of the shaded region is } 30.96\ cm^2.}$

Section-B

2 12) Integers, 1 to 100. (between)

=> total = 98 possible outcomes.

i) divisible by 8 → 12 numbers. (8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96).

=> Probability = $\frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{12}{98} = \boxed{\frac{6}{49}}$

ii) not divisible by 8 => 98 − 12 = 86 numbers.

=> Probability = $\frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{86}{98} = \boxed{\frac{43}{49}}$

~~⇒ Shaded area = Area of square − 4 (Area of quadrant) sq. units~~
~~= 144 − 4 (28.26) sq.cm~~
~~= 144 −~~

11) Two dice tossed together.

2

⇒ Total outcome = 36.

i) doublet: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) → 6 possibilities.

$$\text{Probability} = \frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{6}{36} = \boxed{\frac{1}{6}}.$$

ii) Sum of 10: (4,6), (6,4), (5,5) → 3 possibilities.

$$\text{Probability} = \frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{3}{36} = \boxed{\frac{1}{12}}.$$

9) Sum of first 8 multiples of 3:

2

Forms an AP, $a = 3$, $d = 3$, $n = 8$.

$$\text{Sum} = S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_8 = \frac{8}{2}\left[2 \times 3 + (8-1)3\right] = 4\left[6 + 21\right] = 4 \times 27 = 108.$$

The sum of the first 8 multiples of 3 is $\boxed{108}$

8) Given, rectangle ABCD.

2

$\Rightarrow$ opposite sides are equal.

hence, $x+y=30 \rightarrow ①$

$x-y=14 \rightarrow ②$.

$① + ②$, $2x = 44$

$x = 22$.

Substituting in ①, $22+y=30$

$y=8$.

$\Rightarrow \boxed{x=22,\ y=8}$

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7) Given, $\sqrt{2}$ is irrational

2

To prove: $5+3\sqrt{2}$ is irrational.

Proof: Let us assume $5+3\sqrt{2}$ is rational. So it is in form $\frac{a}{b}$. ~~(a, b ∈ R, b≠0)~~ $[a, b \in Z,\ b \neq 0,\ HCF(a,b)=1]$

$\Rightarrow 5+3\sqrt{2} = \frac{a}{b}$

$3\sqrt{2} = \frac{a}{b} - 5$.

$3\sqrt{2} = \frac{a-5b}{b}$

$\sqrt{2} = \frac{a-5b}{3b}$.

This shows that $\sqrt{2}$ is ~~irrational~~ rational ($a-5b$ and $3b$ are integers).

But we know that $\sqrt{2}$ is irrational.

This contradicts our assumption that $5+\sqrt{2}\times 3$ is rational.

$\Rightarrow$ $5+3\sqrt{2}$ is irrational, hence proved.

Q:19 Points $A(2,3)$, $B(6,-3)$ divided by $P(4,m)$.

Let the ratio be $k:1$.

2

By ~~seg~~ section formula,

$$P(4,m) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$(4,m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$$

$$\Rightarrow \frac{6k+2}{k+1} = 4$$

$$6k+2 = 4k+4$$

$$2k = 2$$

$$k = 1.$$

$\rightarrow$ The ratio is $1:1$.

Now, $m = \frac{-3k+3}{k+1}$

$$m = \frac{-3+3}{1+1}$$

$m = 0.$ $\rightarrow$ $\boxed{\text{Value of } m \text{ is } 0\text{, the point is } P(4,0).}$

Section - A.

6) $\frac{AB}{PQ} = \frac{1}{3}$. $\frac{ar\triangle ABC}{ar\triangle PQR} = \frac{AB^2}{PQ^2} = \frac{1}{3^2} = \frac{1}{9}$.

$\boxed{\text{Ratio of areas is } \frac{1}{9}.}$

5) $\cos^2 67° - \sin^2 23°$

$\cancel{= \cos^2 67° - \cos^2 67°}$

$(\cos 67 + \sin 23)(\cos 67 - \sin 23)$

$= (\cos 67 + \cos 67)(\cos 67 - \cos 67)$ $[\cos\theta = \sin(90-\theta)]$.

$= 0$.

$\boxed{\text{The value is } 0}$.

4) $d = -4$, $a_7 = 4$. $\boxed{\text{The first term is } 28.}$

$t_n = a + (n-1)d$.

$t_7 = a + (7-1)(-4)$

$4 = a + 6 \times (-4)$

$a = 24 + 4$

$a = 28$.

3) Distance between (x, y) and $(0,0)$.

$\Rightarrow \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{(x-0)^2+(y-0)^2}$

$= \sqrt{x^2+y^2}$.

$\boxed{\text{The distance is } \sqrt{x^2+y^2}.}$

1

2) smallest prime = 2

smallest composite = 4

HCF $(2,4) = 2$.

$\boxed{\text{The HCF of the smallest prime and smallest composite is 2.}}$

1

1) $x^2-2kx-6=0$. let α be other root.

Product $= \frac{c}{a} = \frac{-6}{1} = -6$.

$3 \times \alpha = -6$

$\alpha = -2$.

Sum $= \frac{-b}{a} = -\frac{(-2k)}{1} = 2k$.

$\Rightarrow 3+(-2) = 2k$

$1 = 2k$, $k = \frac{1}{2}$.

$\boxed{\text{Value of } k \text{ is } \frac{1}{2}}$

1